22485 Medical Imaging systems

Lecture 8: Velocity imaging using ultrasound

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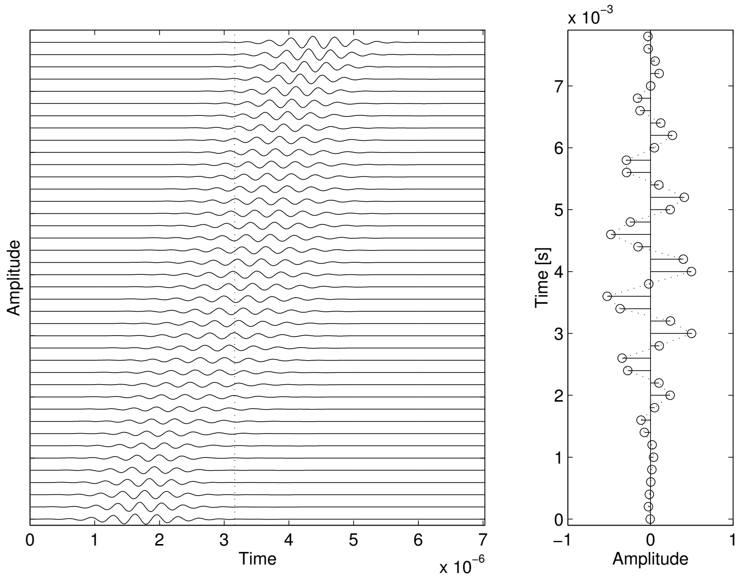
Topic of today: Velocity color flow imaging

1. Important concepts from last lecture
2. Assignment from last lecture
3. Velocity estimation using autocorrelation
   (a) Phase shift estimator
   (b) Stationary echo canceling
4. Velocity estimation using cross-correlation
   (a) Cross-correlation estimator
   (b) Stationary echo canceling
   (c) Implementation and artifacts
5. Exercise 3 on flow simulation
6. Hand-out of ultrasound assignments

Reading material: JAJ, ch. 7 and 8.

A simple model - single scatterer



Signal from a single moving scatterer crossing a beam from a concave transducer.

Time shift between received signals:

$$t_s = \frac{2v_z}{c} T_{prf}$$

Received demodulated signal:

$$\begin{aligned} r_s(i) &= -a \exp(j 2\pi \frac{2v_z}{c} f_0 T_{prf} i - \phi) \\ \phi &= 2\pi f_0 \left(t_z - \frac{2d}{c} \right) \end{aligned}$$

Frequency of received signal:

$$f_p = \frac{2v_z}{c} f_0$$

Discussion on flow estimation system

Calculate what you would get in a velocity estimation system for the phase shift and the power density spectrum for plug flow and parabolic flow.

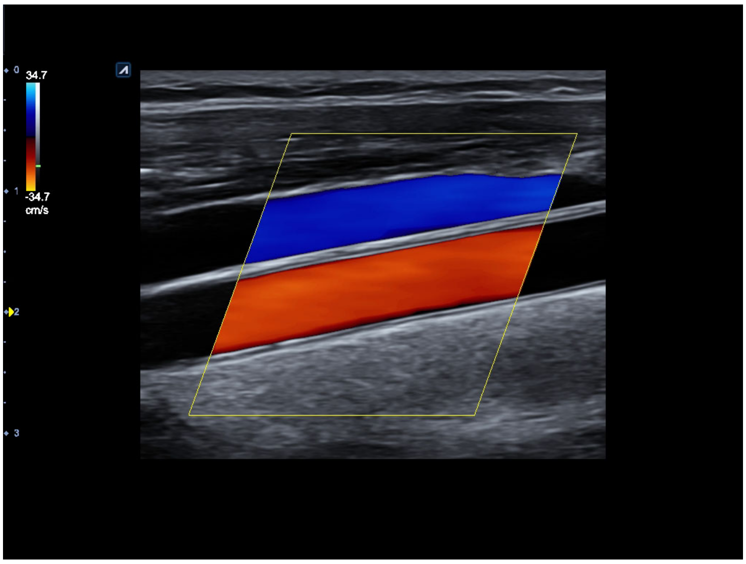
Assume a peak velocity of 0.75 m/s at an angle of 45 degrees at the center of the vessel. The center frequency of the probe is 3 MHz, and the pulse repetition frequency is 10 kHz. The speed of sound is 1500 m/s.

1. How much is the phase shift between two ultrasound pulse emissions?

2. What would the spectrum of the received signal be, if the velocity profile is parabolic?

3. What would the spectrum of the received signal be, if plug flow was found in the vessel?

Color flow map



Blood supply to and from the brain (Carotid artery and jugular vein)

Color flow mapping using phase shift estimation

Received demodulated signal:

$$\begin{aligned} r_{cfm}(i) &= a \cdot \exp(-j(2\pi \frac{2v_z}{c} f_0 i T_{prf} + \phi_f)) \\ &= a \cdot \exp(-j\phi(t)) = x(i) + jy(i) \end{aligned}$$

Velocity estimation:

$$\frac{d\phi}{dt} = \frac{d\left(-2\pi \frac{2v_z}{c} f_0 t + \phi\right)}{dt} = -2\pi \frac{2v_z}{c} f_0$$

Find the change is phase as a function of time gives quantity proportional to the velocity.

Realization

$$\begin{aligned}\tan(\Delta\phi) &= \tan\left(\arctan\left(\frac{y(i+1)}{x(i+1)}\right) - \arctan\left(\frac{y(i)}{x(i)}\right)\right)\\ &= \frac{\frac{y(i+1)}{x(i+1)} - \frac{y(i)}{x(i)}}{1 + \frac{y(i+1)}{x(i+1)} \cdot \frac{y(i)}{x(i)}}\\ &= \frac{y(i+1)x(i) - x(i+1)y(i)}{x(i+1)x(i) + y(i+1)y(i)}\end{aligned}$$

using that

$$\tan(A-B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}.$$

Then

$$\arctan\left(\frac{y(i+1)x(i) - x(i+1)y(i)}{x(i+1)x(i) + y(i+1)y(i)}\right) = -2\pi f_0 \frac{2v_z}{c} T_{prf}.$$

Color flow mapping using phase shift estimation

Using the complex autocorrelation:

$$\begin{aligned}R(m) &= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{i=-N}^{N} r^*_{cfm}(i) r_{cfm}(i+m)\\ &= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{i=-N}^{N} (x(i) - jy(i))(x(i+m) + jy(i+m))\\ &= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{i=-N}^{N} (x(i+m)x(i) + y(i+m)y(i)) + j(y(i+m)x(i) - x(i+m)y(i))\end{aligned}$$

Actual determination from the complex autocorrelation ($m = 1$):

$$v_z = -\frac{cf_{prf}}{4\pi f_0} \arctan\left(\frac{\sum_{i=0}^{N_c-2} y(i+1)x(i) - x(i+1)y(i)}{\sum_{i=0}^{N_c-2} x(i+1)x(i) + y(i+1)y(i)}\right) = -\frac{cf_{prf}}{4\pi f_0} \arctan\left(\frac{\Im\{R(1)\}}{\Re\{R(1)\}}\right)$$

Phase shift estimation with RF sample averaging

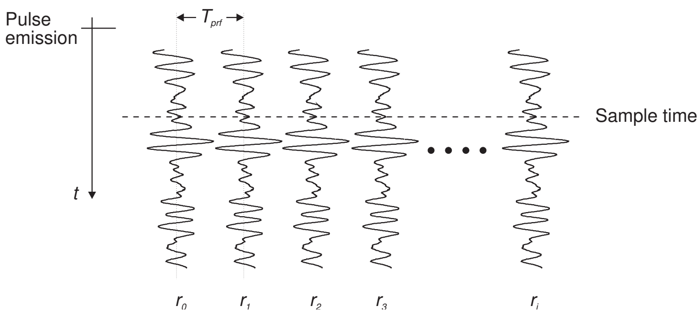
Averaging of RF samples:

$$v_z = -\frac{c f_{prf}}{4\pi f_0} \arctan\left(\frac{\sum_{n=0}^{N_s-1} \sum_{i=0}^{N_c-2} y(n,i+1)x(n,i) - x(n,i+1)y(n,i)}{\sum_{n=0}^{N_s-1} \sum_{i=0}^{N_c-2} x(n,i+1)x(n,i) + y(n,i+1)y(n,i)} \right)$$

Taking samples over a pulse length can improve the estimate, assuming the velocity is roughly constant.

$x(n,i)$	RF sample for time index n and emission number i (in-phase component)
$y(n,i)$	Quadrature component
f_{prf}	Pulse repetition frequency
f_0	Center frequency of transducer
N_s	Number of samples for one pulse length
N_c	Number of emissions
c	Speed of sound

Stationary echo canceling



Canceling:

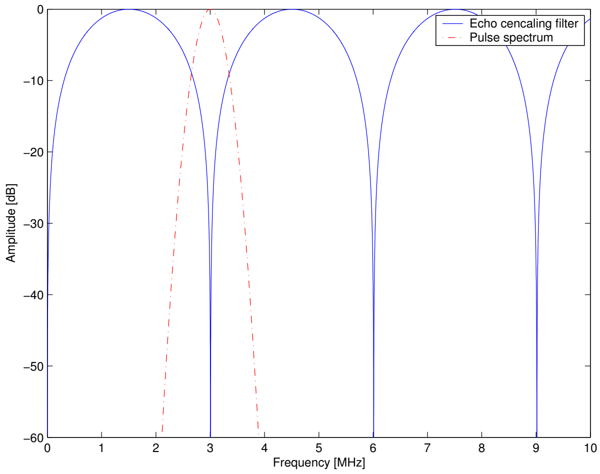
$$\begin{aligned} r_c(i) &= \frac{1}{2}(r(i-1) - r(i)) \\ y_i(t) &= y_{i-1}(t - t_s), \quad t_s = \frac{2v_z}{c} T_{prf} \\ y_c(t) &= \frac{1}{2}(y_{i-1}(t) - y_{i-1}(t - t_s)) \leftrightarrow Y_c(f) = \frac{1}{2} Y_{i-1}(f)(1 - e^{-j2\pi f t_s}) \end{aligned}$$

Transfer function of filter: $|H(f)| = \frac{1}{2}|1 - e^{-j2\pi f t_s}| = |\sin(\pi f \frac{2v_z}{c} T_{prf})|$

Zeros at: $f \frac{2v_z}{c} T_{prf} = p$, Corresponds to: $f = p \frac{c}{2v_z} f_{prf}$

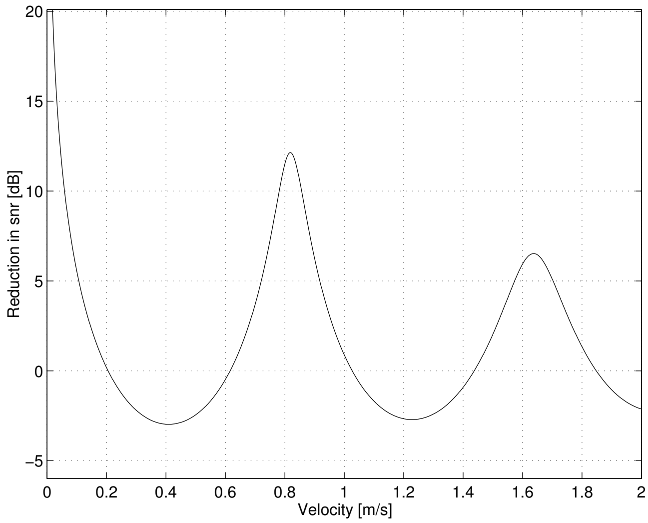
Transfer function of stationary echo canceling filter

$$H(f) = |\sin(\pi f \frac{2v_z}{c} T_{prf})|$$



f_0 = 3 MHz	Center frequency of transducer	f_{prf} = 3.2 kHz	Pulse repetition frequency
B_r = 0.08	Relative bandwidth of pulse	v=0.82 m/s	Blood velocity

Reduction in signal-to-noise ratio



f_0 = 3 MHz	Center frequency of transducer
f_{prf} = 3.2 kHz	Pulse repetition frequency
B_r = 0.08	Relative bandwidth of pulse

Reduction in signal-to-noise ratio due to filter

$$R_{\text{snr}} = \frac{\text{snr}}{\text{snr}_f} = \frac{\sqrt{\dfrac{E[\{p(t)*s_c(t)\}^2]}{E[n^2(t)]}}}{\dfrac{1}{\sqrt{2}}\sqrt{\dfrac{E[\{p(t)*h(t;t_s)*s_c(t)\}^2]}{E[n^2(t)]}}} = \sqrt{2}\sqrt{\frac{E[\{p(t)*s_c(t)\}^2]}{E[\{p(t)*h(t;t_s)*s_c(t)\}^2]}}.$$

For subtraction canceler and Gaussian pulse:

$$\begin{aligned}
R_{\text{snr}} &= \sqrt{\frac{2\sqrt{2}+\exp(-\frac{2}{B_r^2})}{2\sqrt{2}+\exp(-\frac{2}{B_r^2})\xi_1 - 2\sqrt{2}\xi_2\cos(2\pi\frac{f_0}{f_{sh}})}} \\
\xi_1 &= 1-\exp\left(-\frac{1}{2}\left(\frac{\pi B_r f_0}{f_{sh}}\right)^2\right) \qquad \xi_2 = \exp\left(-\left(\frac{\pi B_r f_0}{f_{sh}}\right)^2\right) \\
f_{sh} &= \frac{c}{2v_z} f_{prf}
\end{aligned}$$

$p(t)$	Ultrasound pulse	$S_c(t)$	Signal from blood,
$n(t)$	Measurement noise	$h(t;t_s)$	Impulse response of filter,
B_r	Relative bandwidth of Gaussian pulse	f_0	Center frequency of pulse

General case

Ratio is:

$$R_{\text{snr}} = \frac{\text{snr}}{\text{snr}_f} = \frac{\sqrt{\dfrac{R_{yy}(0)}{R_{nn}(0)}}}{\sqrt{\dfrac{R_{xx}(0)}{R_{ff}(0)}}} = \sqrt{\frac{R_{yy}(0)}{R_{xx}(0)}\frac{R_{ff}(0)}{R_{nn}(0)}}$$

where

$$\begin{aligned}
R_{nn}(0) &= E[n^2(t)] \\
R_{xx}(0) &= E[\{p(t)*h(t;t_s)*s_c(t)\}^2] \\
R_{ss}(\tau) &= \sigma_{ss}^2\delta(\tau) \\
R_{xx}(\tau) &= \sigma_{ss}^2 \cdot R_{pp}(\tau) * R_{hh}(\tau) \\
R_{yy}(\tau) &= \sigma_{ss}^2 \cdot R_{pp}(\tau) \\
R_{ff}(\tau) &= R_{nn}(\tau) * R_{hh}(\tau)
\end{aligned}$$

Autocorrelation of

$R_{ss}(\tau)$	blood scatterer signal	$R_{yy}(\tau)$	received signal
$R_{nn}(\tau)$	noise	$R_{xx}(\tau)$	filtered received signal
$R_{ff}(\tau)$	filtered noise	$R_{hh}(\tau)$	filter impulse response

Standard deviation of the estimates

$$\sigma_v = \frac{c}{2\sqrt{2}\pi T_{prf} f_0}\sqrt{1 - \frac{|R(T_{prf})|}{R(0)}}$$

For a rectangular envelope pulse:

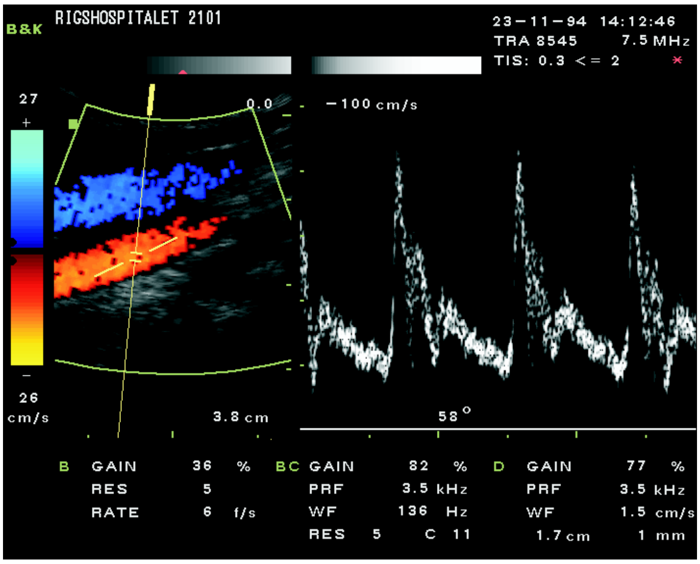
$$\sigma_v = \frac{c}{4\pi f_0}\sqrt{\frac{4}{cT_{prf}T_p}|v_z|} = \sqrt{\frac{c}{4\pi^2 f_0}\frac{f_{prf}}{M}|v_z|}$$

With stationary echo canceling:

$$\sigma_{ve} = \frac{c}{2\sqrt{2}\pi T_{prf} f_0}\sqrt{1 - \frac{|R_{xx}(t_s)|}{R_{xx}(0)}}$$

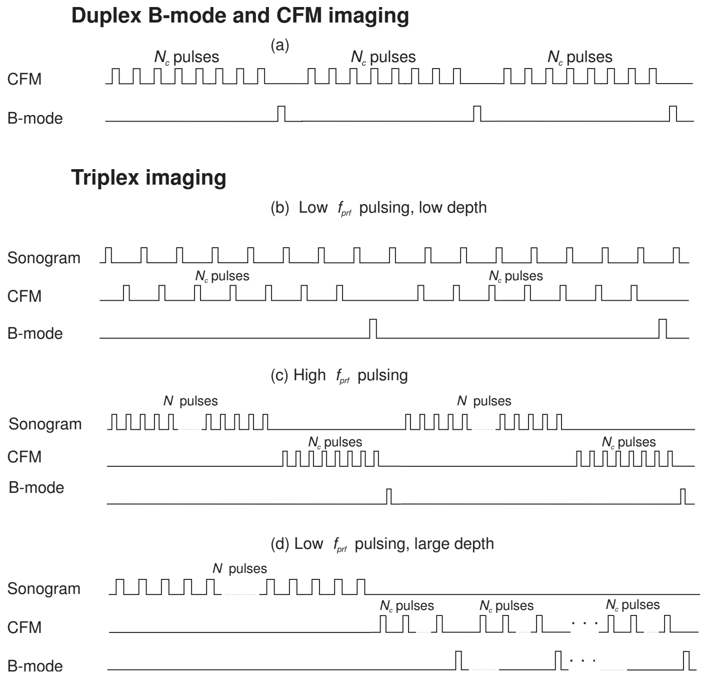
f_0	Center frequency of transducer	c	Speed of sound
v_z	Blood velocity	T_{prf}	Pulse repetition time
M	Cycles in pulse	$\lvert R_{xx}(t_s)\rvert$	Envelope of autocorrelation of received filtered signal

Triplex image

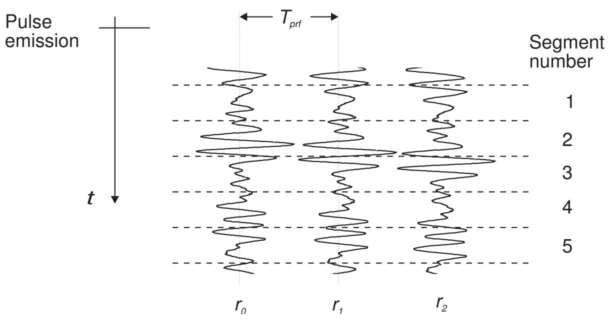


Triplex image of common carotid artery

Emission sequence



Color flow mapping using time shift estimation



Segmentation of RF data prior to cross-correlation

Time shift:

$$t_s = \frac{2\Delta z}{c} = \frac{2|\vec{v}|\cos(\theta)}{c} T_{prf}.$$

Cross-correlation estimator

The signals are related by:

$$r_{s2}(t_2) = r_{s1}(t_2 - T_{prf} - t_s) = r_{s1}(t_1 - t_s)$$

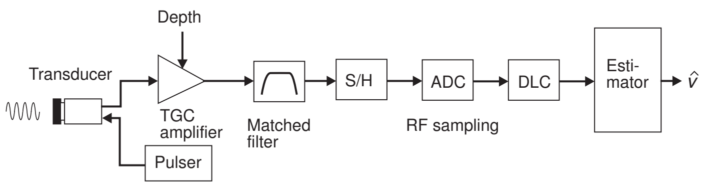
Cross-correlation yields

$$\begin{aligned} R_{12}(\tau) &= \frac{1}{2T}\int_T r_{s1}(t) r_{s2}(t+\tau)dt = \frac{1}{2T}\int_T r_{s1}(t) r_{s1}(t - t_s + \tau)dt \\ &= R_{11}(\tau - t_s) \\ R_{12}(\tau) &= R_{pp}(\tau) * \sigma_s^2 \delta(\tau - t_s) = \sigma_s^2 R_{pp}(\tau - t_s) \end{aligned}$$

The velocity estimate is:

$$\widehat{v}_z = \frac{c}{2}\frac{\widehat{t}_s}{T_{prf}}.$$

Cross-correlation system



Calculation of the cross-correlation:

$$\widehat{R}_{12d}(n, i_{seg}) = \frac{1}{N_s(N_c - 1)} \sum_{i=0}^{N_c-2} \sum_{k=0}^{N_s-1} r_{s_i}(k + i_{seg}N_s) r_{s_{i+1}}(k + i_{seg}N_s + n).$$

Largest detectable velocity:

$$v_{max} = \frac{l_g}{T_{prf}} = \frac{c}{2} N_s \frac{f_{prf}}{f_s}.$$

Minimum velocity

Minimum velocity due to time quantization:

$$v_{min} = \frac{c}{2}\frac{f_{prf}}{f_s}$$

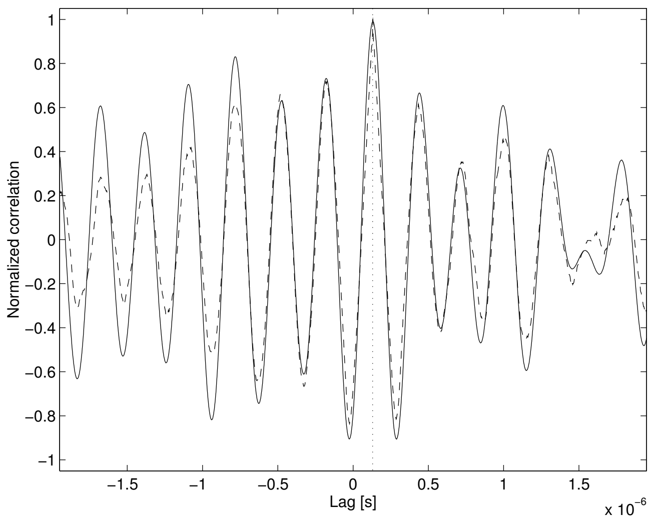
Interpolated peak by polynomial fit:

$$n_{int} = n_m - \frac{\hat{R}_{12d}(n_m+1) - \hat{R}_{12d}(n_m-1)}{2(\hat{R}_{12d}(n_m+1) - 2\hat{R}_{12d}(n_m) + \hat{R}_{12d}(n_m-1))}$$

Interpolated estimate:

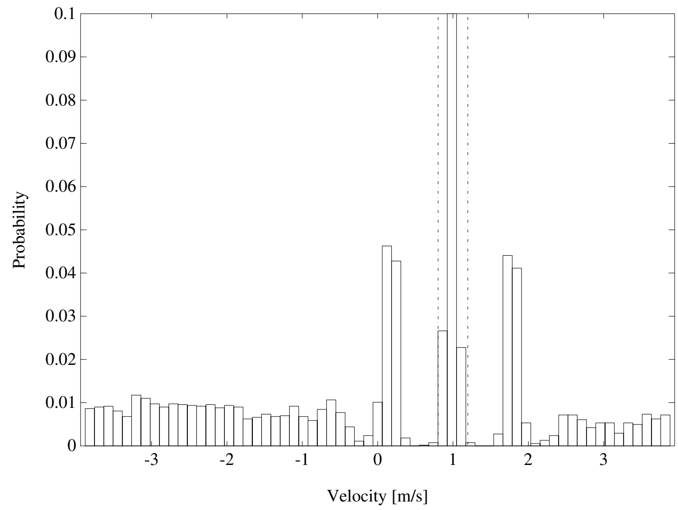
$$\hat{v}_{int} = \frac{c}{2}\frac{n_{int} f_{prf}}{f_s}.$$

Cross-correlation function



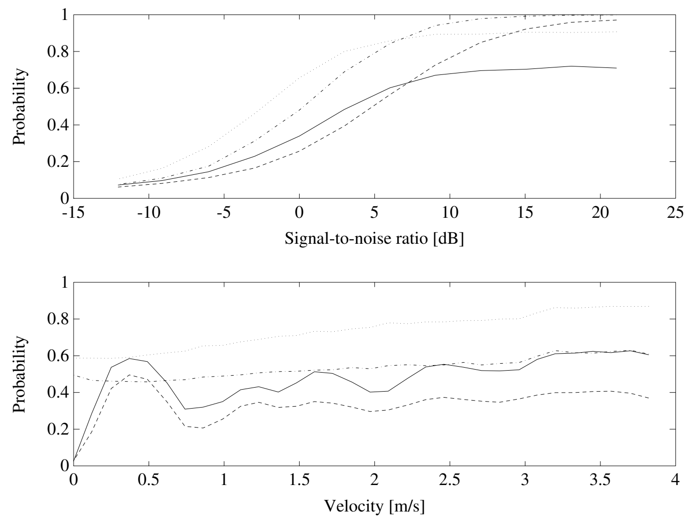
Estimates of cross-correlation using full precision data values (—)
Sign of the data (- - -).

False peak detection



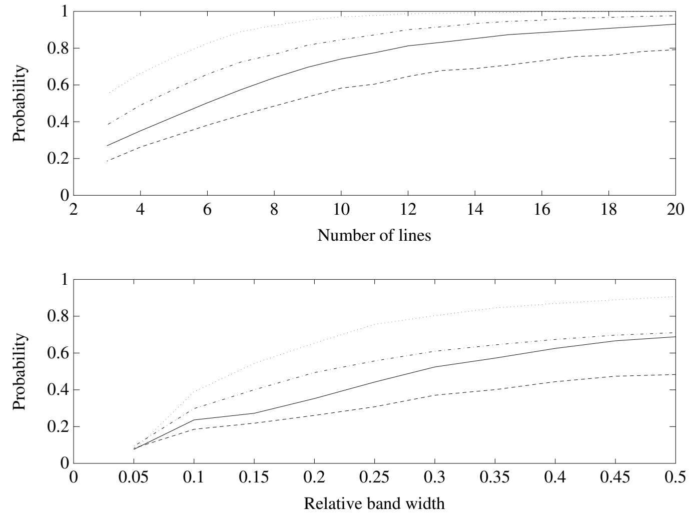
Distribution of velocity estimates (enlarged view). The true velocity is 1 m/s and snr = 0 dB. The peak at v=1 m/s goes to 0.433.

Detection probabilities



Variation in probability of correct detection due to different values of the parameters. — is when full precision data and echo canceling are used, - - - is the sign and echo canceling, $\cdots$ is full precision data without echo canceling, and ·—·— is sign data without echo canceling.

Detection probabilities



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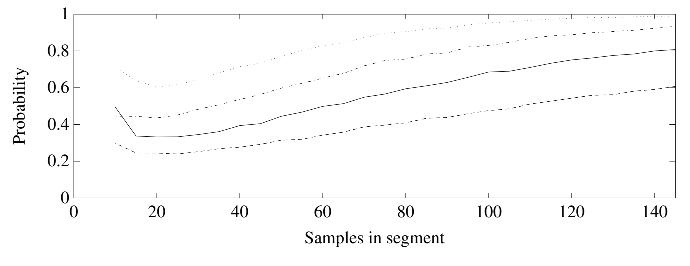
Detection probabilities



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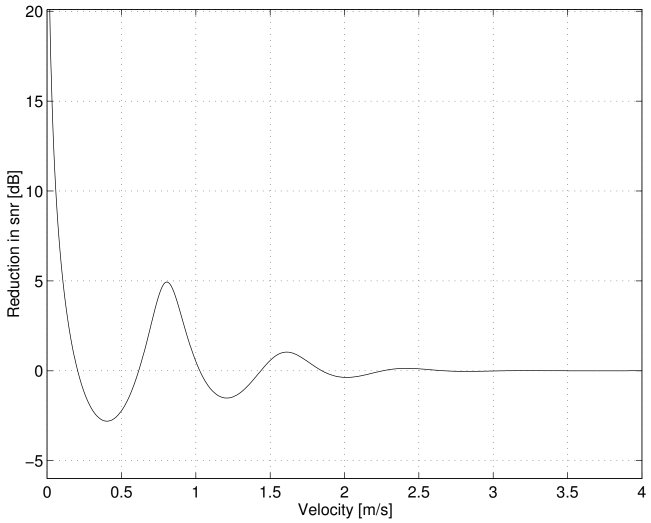
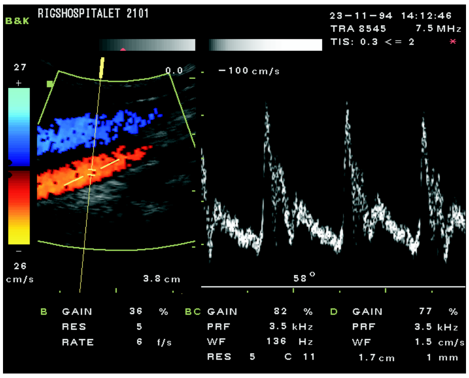
Stationary echo canceling



Reduction of the signal-to-noise ratio due to the stationary echo canceling filter as a function of velocity. A Gaussian 3 MHz pulse with a relative bandwidth of 0.2 was used. The pulse repetition frequency was 3.2 kHz.

Ultrasound systems for velocity imaging

- Two different color flow mapping systems:
  - Autocorrelation systems the velocity from the phase shift between emissions
  - Cross-correlation systems find the velocity from the time shift
- Stationary echo canceling has an influence on SNR
- Time shift system can find larger velocities, but also have a probability for error
- Next time: Simulations and non-linear imaging, chapters. 2.5-6 and 4.2, Pages 27-44 and 70-75
- Research on ultrasound imaging and velocity estimation
- Now: Assignment for next lecture, exercise 3



Discussion for next time on time and phase shift systems

Calculate what you would get in a time and phase shift velocity estimation systems for the parameters given below.

Assume a peak velocity of 0.6 m/s at an angle of 60 degrees at the center of the vessel. The center frequency of the probe is 3 MHz, and the pulse repetition frequency is 3.2 kHz. The speed of sound is 1500 m/s. A Gaussian pulse with a relative bandwidth of 0.2 is used for the cross-correlation system and $B_r = 0.08$ for the autocorrelation system.

1. How much is the time shift between two ultrasound pulse emissions?
2. What is the largest velocity detectable, if the cross-correlation function is calculated and searched over two wavelengths?
3. What is the highest detectable velocity for a phase shift system?
4. What is the loss in SNR for a velocity of 0.05 m/s based on Figures 7.5 and 8.3 for the two systems?

Exercise 3 about generating ultrasound RF flow data

Basic model, first emission:

$$r_1(t) = p(t) * s(t)$$

$s(t)$ - Scatterer amplitudes (white, random, Gaussian)

Second emission:

$$r_2(t) = p(t) * s(t - t_s) = r_1(t - t_s)$$

Time shift t_s:

$$t_s = \frac{2v_z}{c} T_{prf}$$

$r_1(t)$	Received voltage signal	$p(t)$	Ultrasound pulse
$*$	Convolution	v_z	Axial blood velocity
c	Speed of sound	T_{prf}	Time between pulse emissions

Signal processing

1. Find ultrasound pulse (load from file)
2. Make scatterers
3. Generate a number of received RF signals
4. Study the generated signals
5. Compare with simulated and measured RF data