## 22485 Medical Imaging systems

Lecture 3: Ultrasound focusing and modeling

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## Topic of today: Imaging in ultrasound

## Focusing and spatial impulse responses

1. Clinical study
2. Go through signal processing quiz
3. Repetition and assignment from last
4. Arrays and focusing using delay-and-sum
5. Ultrasound fields and resolution
6. Questions for exercise 1

Reading material: JAJ, ch. 2., p. 24-36

Self-study: CW fields

**Signal processing quiz**

1. What is the equation for the spectrum of a square wave signal with a duration of $T$ seconds and an amplitude of $a$?

   (a) At what frequency is the first zero in the spectrum?

   (b) What is the amplitude of the spectrum at a frequency of 0 Hz?

2. What is the spectrum of a tone pulse given by: $g(t) = a(t)\,\mathrm{sin}(2\pi f0t)$, where $a(t) = 1$ for $0 < t < M/f0$ and zero elsewhere.

3. Make a sketch of the amplitude spectrum of an $M$ cycle sinus pulse with a frequency of $f_0$.

4. What is the autocorrelation function of a stochastic, white signal?



5. Make a drawing of the output from the following Matlab program:

```
M=4;
f0=5e6;
T=M/f0;
fs=8*f0;
pulse=2*sin(2*pi*f0*(0:1/fs:T));
plot(pulse)
```

6. A signal is generated by $y(n) = h(n) * e(n)$, where $h(n)$ is the impulse response of a filter and $e(n)$ is a stochastic, white signal. Express the autocorrelation function of $y(n)$ from the autocorrelation functions $R_h(k)$ of $h(n)$ and $R_e(k)$ of $e(n)$.

## From last lecture: The wave equation

Speed of sound $c$:

$$c = \sqrt{\frac{1}{\rho\kappa}}$$

Linear wave equation:

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

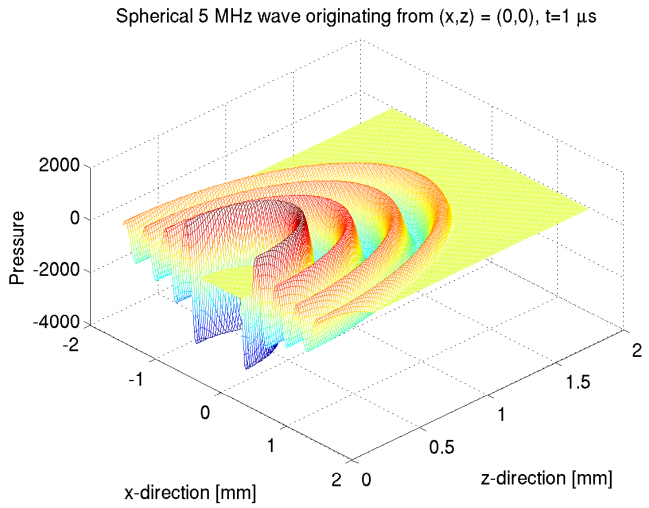
General solutions:

$$p(t,x) = g(t \pm \frac{x}{c})$$

$\frac{x}{c}$ is time delay



## Spherical wave




$$p(t,\vec{r}) = \frac{p_0}{|\vec{r}|}\sin(2\pi f_0(t - \frac{|\vec{r}|}{c}))$$

Propagates radially from a center point.

Used for describing diffraction, focusing, and ultrasound fields.

## Attenuation of ultrasound

| Tissue | Attenuation dB/[MHz·cm] |
|---|---|
| Liver | 0.6 − 0.9 |
| Kidney | 0.8 − 1.0 |
| Spleen | 0.5 − 1.0 |
| Fat | 1.0 − 2.0 |
| Blood | 0.17 − 0.24 |
| Plasma | 0.01 |
| Bone | 16.0 − 23.0 |

*Approximate attenuation values for tissue penetrated for human tissue*

Pulse-echo: depth of 10 cm, $f_0 = 5$ MHz, attenuation 0.7 dB/[MHz·cm]:

Attenuation: $\mathbf{2} \cdot 10 \cdot 5 \cdot 0.7 = 70$ dB



## Amplitude attenuation transfer function for a plane wave

$$|H(f; z)| = \exp(-(\beta_0 z + \beta_1 f z)), \tag{1}$$

$z$ - depth in tissue
$f$ - frequency
$\beta_0$ - frequency-independent attenuation
$\beta_1$ - frequency-dependent term expressed in Np/[MHz·cm] (Np - Nepers)

Converted to Nepers by dividing with 8.6859

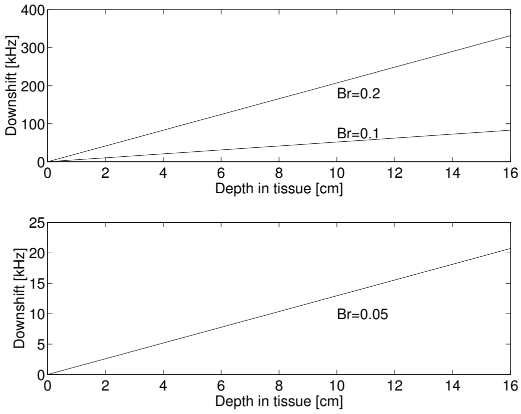
0.7 dB/[MHz·cm] is 0.0806 Np/[MHz·cm].

Frequency dependent term is the major source of attenuation

Frequency independent term is often left out.

## Effect of attenuation

Down-shift in center frequency: $f_{mean} = f_0 - (\beta_1 B_r^2 f_0^2) z$




| | |
|---|---|
| $f_0$ | Center frequency of transducer |
| $c$ | Speed of sound |
| $v$ | Blood velocity |
| $\beta_1$ | Frequency dependent attenuation |
| $B_r$ | Relative bandwidth of pulse |

$f_0 = 3$ MHz, $\beta_1 = 0.5$ dB/[MHz·cm]



## FDA safety limits

| Use | $I_{spta.3}$ mW/cm$^2$ In Situ | $I_{spta.3}$ mW/cm$^2$ Water | $I_{sppa.3}$ W/cm$^2$ In Situ | $I_{sppa.3}$ W/cm$^2$ Water | $I_m$ W/cm$^2$ In Situ | $I_m$ W/cm$^2$ Water | MI |
|---|---|---|---|---|---|---|---|
| Cardiac | 430 | 730 | 190 | 240 | 160 | 600 | 1.90 |
| Peripheral vessel | 720 | 1500 | 190 | 240 | 160 | 600 | 1.90 |
| Ophthalmic | 17 | 68 | 28 | 110 | 50 | 200 | 0.23 |
| Fetal imaging (a) | 94 | 170 | 190 | 240 | 160 | 600 | 1.90 |

Highest known acoustic field emissions for commercial scanners as stated by the United States FDA (The use marked (a) also includes intensities for abdominal, intra-operative, pediatric, and small organ (breast, thyroid, testes, neonatal cephalic, and adult cephalic) scanning) from the the September 9, 2008, FDA Guidance.

**Discussion from last**

Use the ultrasound system from the first discussion assignment

1. What is the largest pressure tolerated for cardiac imaging?

2. What displacement will this give rise to in tissue?



**Discussion assignment from last time**

B-mode system, 10 cm penetration, 300 $\lambda$

Frequency 4.5 MHz, one cycle emission.

Plane wave with an intensity of 730 mW/cm$^2$ (cardiac, water)

$z = 1.48 \cdot 10^6$ kg/[m$^2$ s]

Pulse emitted: $f_0 = 4.5$ MHz, $M = 1$ period, $f_{prf} = 7.5$ kHz, $T_{prf} = 133\mu$s

Pressure emitted:

$$\begin{aligned} I_{spta} &= \frac{1}{T_{prf}} \int_0^{M/f_0} I_i(t, \vec{r}) dt = \frac{M/f_0}{T_{prf}} \frac{P_0^2}{2z} \\ p_0 &= \sqrt{\frac{I_{spta} 2z T_{prf}}{M/f_0}} = \sqrt{\frac{0.730 \cdot 10^4 \cdot 2 \cdot 1.48 \cdot 10^6 \cdot 133 \cdot 10^{-6}}{1/4.5 \cdot 10^6}} = 3.60 \cdot 10^6 \text{ Pa} = 36.0 \text{ atm} \end{aligned}$$

The normal atmospheric pressure is 100 kPa.

**Example continued**

The particle velocity is $U_0 = \frac{p_0}{Z} = \frac{3.60 \cdot 10^6}{1.48 \cdot 10^6} = 2.43$ m/s.

The particle velocity is the derivative of the particle displacement, so

$$z(t) = \int U_0 \sin(\omega_0 t - kz) dt = \frac{U_0}{\omega_0} \cos(\omega_0 t - kz).$$

The displacement is

$$z_0 = U_0/\omega_0 = 2.43/(2\pi \cdot 4.5 \cdot 10^6) = 86 \text{ nm}.$$

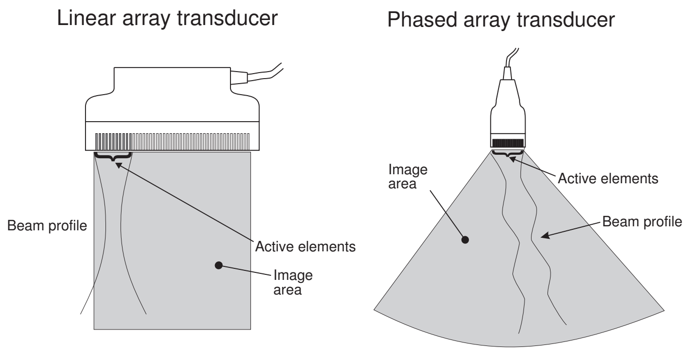


# How do we use ultrasound arrays?

## Ultrasound imaging using arrays

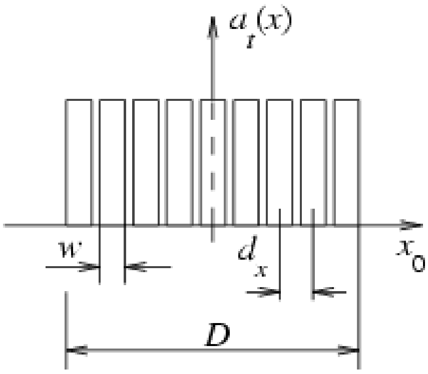
- Multi-element transducer arrays are used:
  - Linear arrays - element width $\approx \lambda$
  - Phased arrays - element width $\approx \lambda/2$

1. Focused transmit by applying different delays on elements
2. Field propagates along beam direction. Echoes scattered back.
3. Received signals are delayed and summed (beamformed). Delays change as a function of time (dynamic delays).
4. The same process is repeated for another image direction.





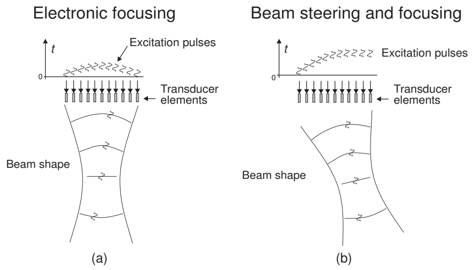

## Array geometry




- $d_x$ - Element pitch. For linear array: $\approx \lambda = c/f_0$, for phased array: $\approx \lambda/2$
- $w$ - Width of element
- $k_e = d_x - w$ - Kerf (gap between elements)
- $D = (N_e - 1)d_x + w$ - Size of transducer

- Commercial 7 MHz linear array:
  - Elements: $N_e = 192$ ,
    64 active at the same time
  - $\lambda = c/f_0 = 1540/7 \cdot 10^6 = 0.22$ mm
  - Pitch: $d_x = 0.208$ mm
  - Width: $D = 3.9$ cm
  - Height: $h = 4.5$ mm
  - Kerf: $k_e = 0.035$ mm

## Focusing and beamforming




Time from the center of element to field point:

$$t_i = \frac{1}{c}\sqrt{|\vec{r}_i - \vec{r}_f|^2}$$

$\vec{r}_f$ - position of the focal point
$\vec{r}_i$ - center of physical element number $i$

Reference point on aperture (defines time $t = 0$):

$$t_c = \frac{1}{c}\sqrt{|\vec{r}_c - \vec{r}_f|^2}$$

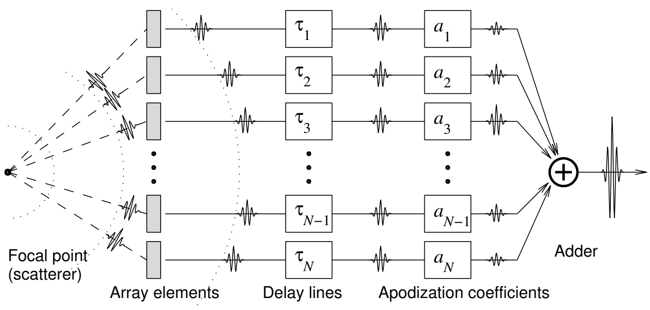
$\vec{r}_c$ - reference center point on the aperture.

Delay to use on each element of the array:

$$\Delta t_i = \frac{1}{c}\left(\sqrt{|\vec{r}_c - \vec{r}_f|^2} - \sqrt{|\vec{r}_i - \vec{r}_f|^2}\right)$$



## Beamforming in Modern Scanners




$$s(t) = \sum_{1}^{N_{xdc}} a_i y_i(t - \tau_i)$$

$$\tau_i = \frac{|\vec{r}_c - \vec{r}_f| - |\vec{r}_i - \vec{r}_f|}{c}$$

- $a_i$ - Weighting coefficient (apodization)
- $y_i(t)$ - Received signal
- $\vec{r} = [x, y, z]^T$ - Spatial position
- $\vec{r}_i$ - Position of transducer element,
- $\vec{r}_c$ - Beam reference point
- $\vec{r}_f$ - Focal point
- $c$ - Speed of sound

## Focusing and apodization time lines

Focusing:

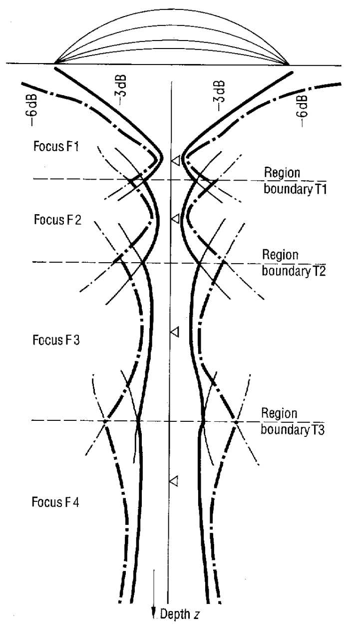
| From time | Focus at |
|---|---|
| 0 | $\vec{r}_1$ |
| $t_1$ | $\vec{r}_1$ |
| $t_2$ | $\vec{r}_2$ |
| $\vdots$ | $\vdots$ |

Purpose: to maintain a roughly constant F-number $F_{\#} = \frac{z}{D}$

Lateral resolution at focus $\approx \lambda F_{\#}$

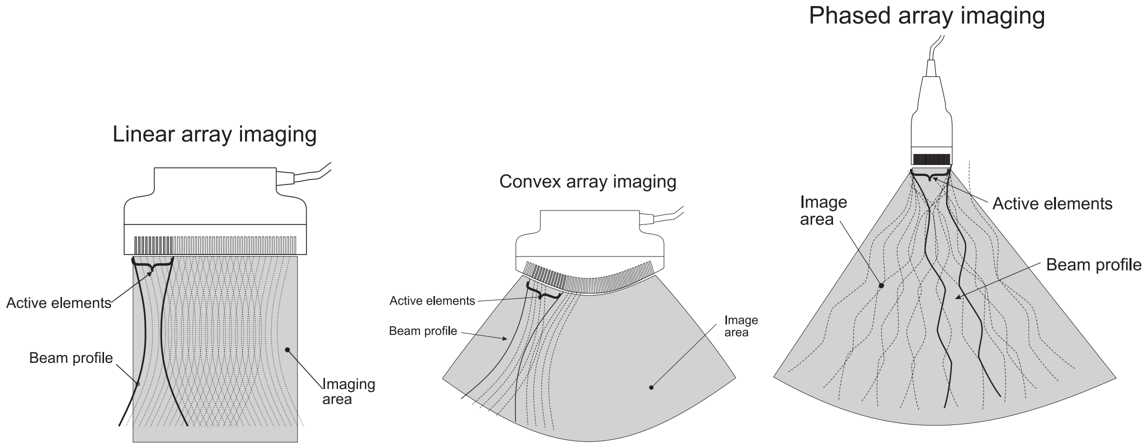
Apodization:

| From time | Apodize with |
|---|---|
| 0 | $a_{1,1}, a_{1,2}, \cdots a_{1,N_e}$ |
| $t_1$ | $a_{1,1}, a_{1,2}, \cdots a_{1,N_e}$ |
| $t_2$ | $a_{2,1}, a_{2,2}, \cdots a_{2,N_e}$ |
| $t_3$ | $a_{3,1}, a_{3,2}, \cdots a_{3,N_e}$ |
| $\vdots$ | $\vdots$ |

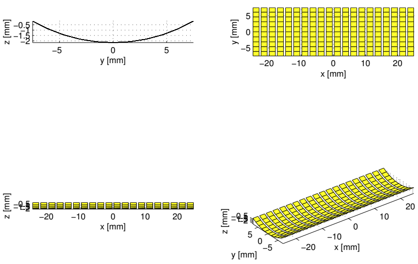


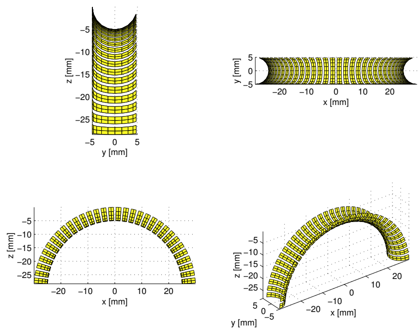



## Imaging methods



## Typical transducers



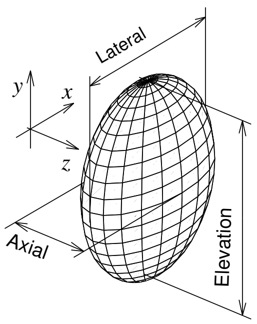


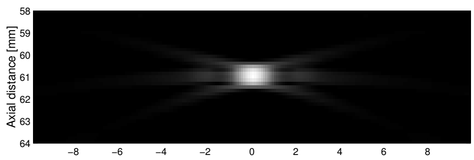
Linear or phased array

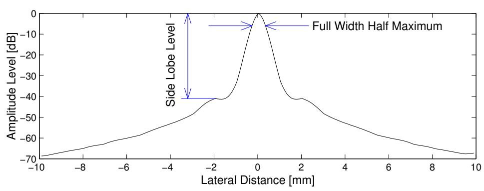
Convex array (curvature exaggerated)



## PSF Characteristics




- The PSF is three dimensional
- The B-mode images are only 2-D
- Displayed on a logarithmic scale
- Maximum taken along $z$
- Parameters used: FWHM, side- and grating-lobe level

- Axial resolution: $R_z = \frac{M}{2}\lambda = \frac{M}{2}\frac{c}{f_0}$ (fixed)
- Lateral resolution: $R_x = \frac{Z}{D}\lambda = F\#_x\lambda$ (dynamic)
- Azimuth (out of plane): $R_y = \frac{Z}{H}\lambda = F\#_y\lambda$ (fixed)

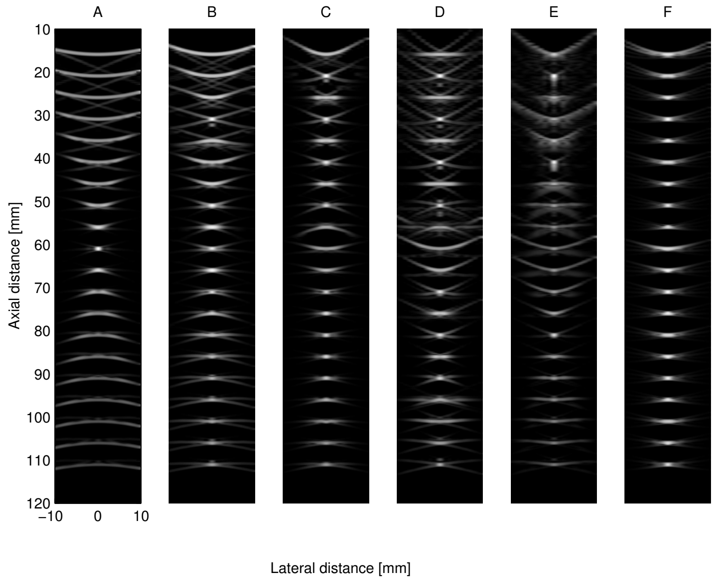
$M$ - Periods in pulse, $\lambda$ - Wavelength, $Z$ - Imaging depth,

$D$ - Active probe width, $H$ - Probe height, $F\#$ - F-number

## System Characterization

A system can be characterized by the *point-spread-function* (PSF). The point spread function is:

$$\mathbf{p}(\vec{x}, t) = v_{pe}(t) \underset{t}{*} h_{pe}(\vec{r}, t).$$



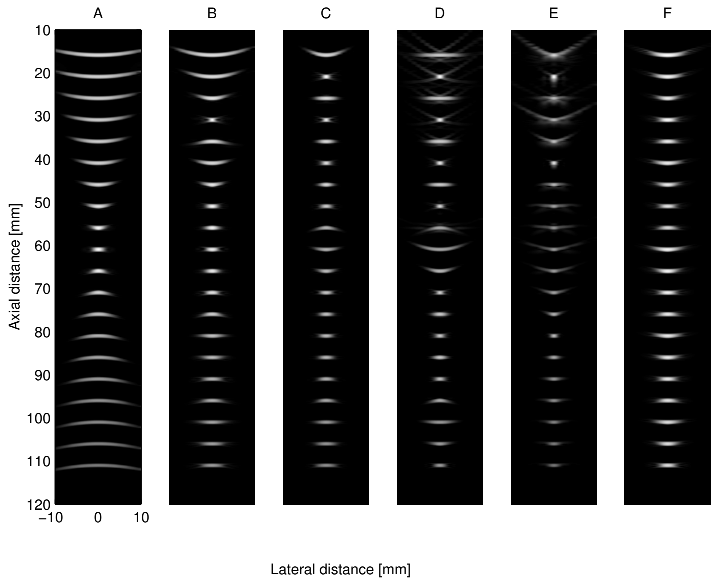

Examples of PSF without apodization:

- A - one focal zone
- B - 6 receive focal zones
- C - 6 xmt and rcv zones
- D - 128 elem, 4 xmt zones, 7 rcv zones
- E - 128 elem, 4 xmt zones, dynamic rcv
- F - 128 elem, xmt $F_{\#} = 4$, rcv $F_{\#} = 2$



## Sidelobe Reduction

The sidelobes can be improved by applying apodization.




Examples of PSF with apodization:

- A - one focal zone
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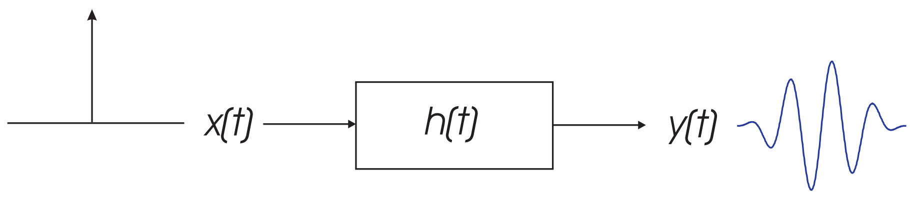
- D - 128 elem, 4 xmt zones, 7 rcv zones
- E - 128 elem, 4 xmt zones, dynamic rcv
- F - 128 elem, xmt $F_{\#} = 4$, rcv $F_{\#} = 2$

## How can we calculate the ultrasound fields?

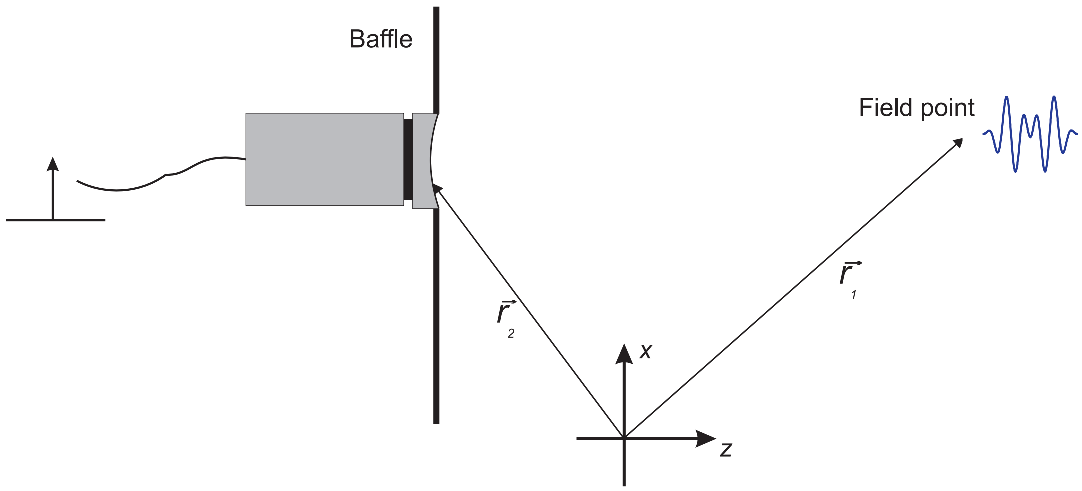


**Linear Electrical System**




Fully characterized by it's impulse response

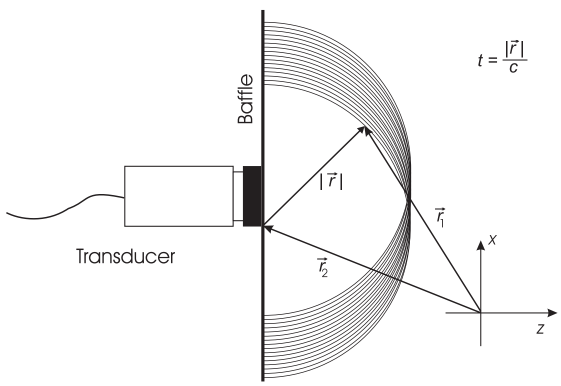
## Linear Acoustic System




Impulse response at a point in space - Spatial Impulse Responses - $h(\vec{r}, t)$



## Huygens' Principle




Arrival times: $t = d/c$, summation of spherical waves

Moving the point results in a new impulse response:

Spatial Impulse Responses - $h$

**Rayleigh's Integral**

$$p(\vec{r}_1, t) = \frac{\rho_0}{2\pi} \int_S \frac{\frac{\partial v_n(\vec{r}_2, t - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{\partial t}}{|\ \vec{r}_1 - \vec{r}_2\ |} d^2\vec{r}_2$$

| | |
|---|---|
| $\mid \vec{r}_1 - \vec{r}_2 \mid$ | - Distance to field point |
| $v_n(\vec{r}_2, t)$ | - Normal velocity of transducer surface |

Summation of spherical waves from each point on the aperture surface



**Spatial impulse response:**

$$h(\vec{r}_1, t) = \int_S \frac{\delta(t - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{2\pi \mid \vec{r}_1 - \vec{r}_2 \mid} dS$$

Emitted field:

$$p(\vec{r}_1, t) = \rho_0 \frac{\partial v(t)}{\partial t} * h(\vec{r}_1, t)$$

Pulse echo field:

$$v_r(\vec{r}_1, t) = v_{pe}(t) * h_{pe}(\vec{r}_1, t) = v_{pe}(t) * h_t(\vec{r}_1, t) * h_r(\vec{r}_1, t)$$

## Ultrasound fields

Emitted field:

$$p(\vec{r}_1, t) = \rho_0 \frac{\partial v(t)}{\partial t} * h(\vec{r}_1, t)$$

Pulse echo field:

$$\begin{aligned} v_r(\vec{r}_1, t) &= v_{pe}(t) * f_m(\vec{r}_1) * h_{pe}(\vec{r}_1, t) \\ &= v_{pe}(t) * f_m(\vec{r}_1) * h_t(\vec{r}_1, t) * h_r(\vec{r}_1, t) \\ f_m(\vec{r}_1) &= \frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c} \end{aligned}$$

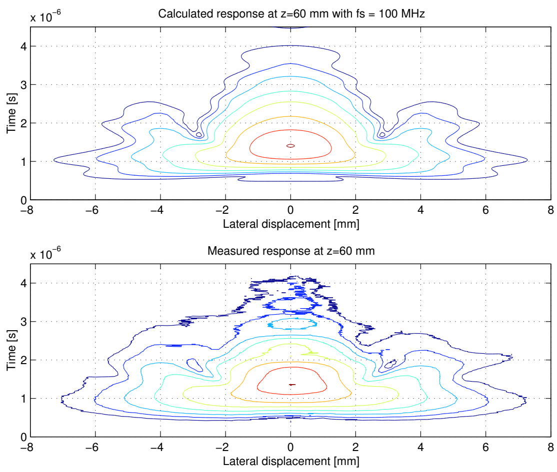
Continuous wave fields:

$$\mathcal{F}\{p(\vec{r}_1, t)\}, \qquad \mathcal{F}\{v_r(\vec{r}_1, t)\}$$

All fields can be derived from the spatial impulse response.



## Point spread functions




Point spread function for concave, focused transducer
Top: simulation top
Bottom: tank measurement (6 dB contour lines)

**Discussion for next time**

What are the focusing delays on an array.

Parameters: 64 element array, 5 MHz center frequency, $\lambda$ pitch, all elements used in transmit

Focusing is performed directly down at the array center.

1. Imaging depth of 1 cm: How much should the center element be delayed?

2. Imaging depth of 10 cm: How much should the center element be delayed?



**Learned today**

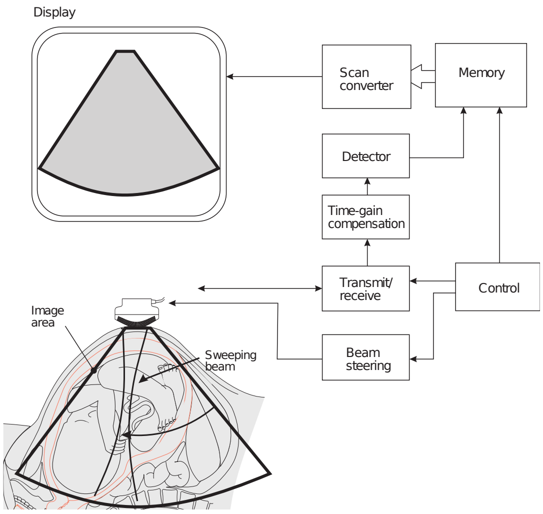
- Focusing of arrays using delay-and-sum beamforming

- Point spread functions

- Calculation of fields using spatial impulse response

Next time: Ch. 2 in JAJ, pages 36-44 on array geometries and their design

## Signal processing in ultrasound system - Exercise 1




Signal processing in a simple ultrasound system with a single element ultrasound transducer.



## Signal processing

1. Acquire RF data (load from file)
2. Perform Hilbert transform to find envelope:
   $\text{env}(n) = |y(n)^2 + \mathcal{H}\{y(n)\}^2|$
3. Compress the dynamic range to 60 dB:
   $\text{log\_env}(n) = 20\log_{10}(\text{env}(n))$
4. Scale to the color map range (128 gray levels)
5. Make polar to rectangular mapping and interpolation
6. Display the image
7. Load a number of clinical images and make a movie

## Polar to rectangular mapping and interpolation 1

Two step procedure to set-up tables and then perform the interpolation.

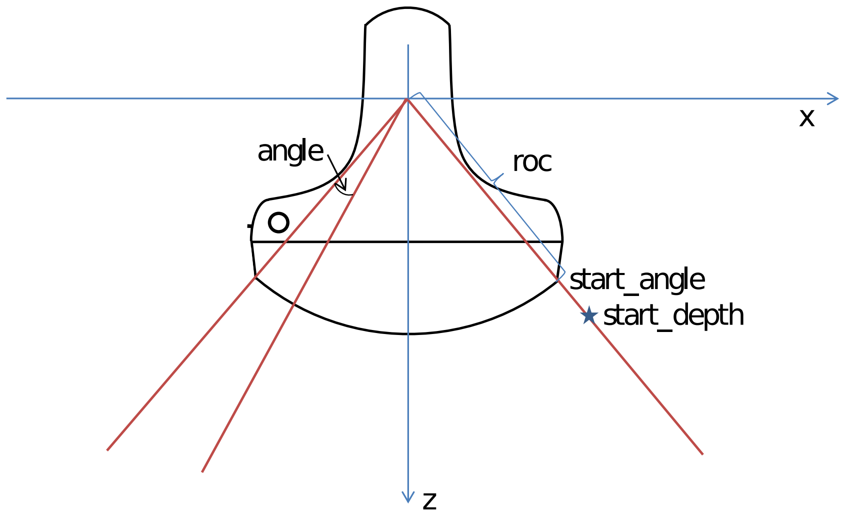
Table setup:

```
%  Function for calculating the different weight tables
%  for the interpolation. The tables are calculated according
%  to the parameters in this function and are stored in
%  the internal C-code.
%
%  Input parameters:
%
%        start_depth    - Depth for start of image in meters
%        image_size     - Size of image in meters
%
%        start_of_data  - Depth for start of data in meters
%        delta_r        - Sampling interval for data in meters
%        N_samples      - Number of samples in one envelope line
%
%        theta_start    - Angle for first line in image
%        delta_theta    - Angle between individual lines
%        N_lines        - Number of acquired lines
```



```
%
%        scaling        - Scaling factor from envelope to image
%        Nz             - Size of image in pixels
%        Nx             - Size of image in pixels
%
%  Output:  Nothing, everything is stored in the C program
%
%  Calling: make_tables (start_depth, image_size,            ...
%                        start_of_data, delta_r, N_samples,   ...
%                        theta_start, delta_theta, N_lines,   ...
%                        scaling, Nz, Nx);
%
%  Version 1.1, 11/9-2001, JAJ: Help text corrected
```

## Geometry




Relation between the different variables

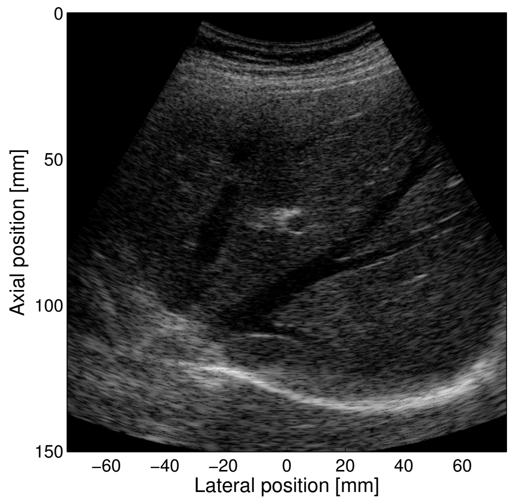


## Polar to rectangular mapping and interpolation 2

Perform the actual interpolation:

```
%  Function for making the interpolation of an ultrasound image.
%  The routine make_tables must have been called previously.
%
%  Input parameters:
%
%         envelope_data - The envelope detected and log-compressed data
%                         as an integer array as 32 bits values  (uint32)
%
%  Output:  img_data     - The final image as 32 bits values
%
%  Calling: img_data = make_interpolation (envelope_data);
```

## Resulting image




Ultrasound image of portal veins in the liver.



## Useful Matlab commands

Loading of files:

```
load (['in_vivo_data/8820e_B_mode_invivo_frame_no_',num2str(j)])
```

Making a movie:

```
for j=1:66

  image(randn(20))
  colormap(gray(256))
  axis image

  F(j)=getframe;
  end

%  Play the movie 5 times at 22 fr/s

movie(F,5, 22)
```