22485 Medical Imaging Systems

Lecture 2: September 2024

Ultrasound Physics

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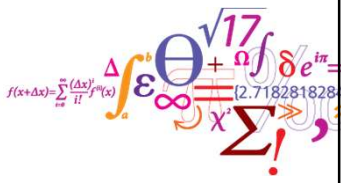

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Outline of today's discussion

1. Discussion assignment on B-mode imaging
2. Signal processing quiz
3. Derivation of wave equation and role of speed of sound
4. Wave types and properties
5. Intensities and safety limits
6. Scattering of ultrasound
7. Questions for exercise 1 and practical details

2

Design an ultrasound B-mode system

Assume that a system can penetrate **300** wavelengths

It should penetrate down to 10 cm in a liver

1. What is the largest pulse repetition frequency possible?
2. What is the highest possible transducer center frequency?
3. What is the axial resolution?
4. What is the lateral resolution for an F-number of 2?

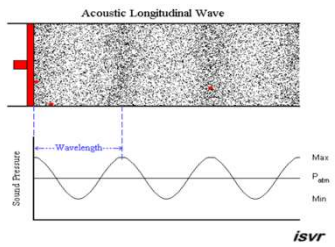
3

Signal processing quiz

1. Takes 15 minutes
2. Hand in anonymously
3. We will discuss it after the break

4

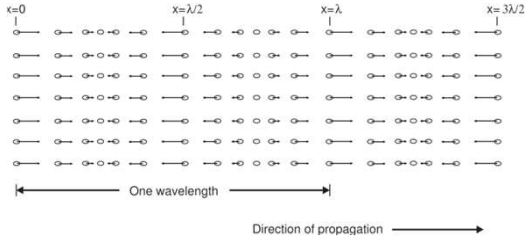
Wave types



Source: Institute of Sound & Vibration Research (UK)

5

Wave types



Disturbance propagates. No transport of mass.

Only longitudinal waves are possible in soft tissue. Shear or transverse waves attenuate very quickly

6

Derivation of the wave equation in one dimension

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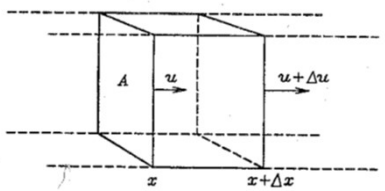
Forces on volume element



P: Acoustic pressure due to wave

u: Particle velocity

Newton's second law:

F = ma

$$-A\Delta p = A\rho\Delta x\frac{\partial u}{\partial t}$$

Δp – Difference in pressure from x to x+Δx. Negative pressure gives larger volume and positive pressure gives smaller volume

Find the limit for dx→0 gives the first equation:

$$\frac{\partial p}{\partial x} = -\delta\frac{\partial u}{\partial t}$$

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Preservation of mass

Change in volume due to change in particle velocity:

$$\Delta \text{vol} = A\Delta u\Delta t$$

Assume linear relation to pressure

$$\Delta \text{vol} = -\kappa(A\Delta x)\Delta p$$

If the pressure on the volume is positive, then the volume gets smaller and the volume change is negative

Combined: $A\Delta u\Delta t = -\kappa(A\Delta x)\Delta p$

After limit gives the second equation: $\frac{\partial u}{\partial x} = -\kappa\frac{\partial p}{\partial t}$

Combining the two equations

$$\frac{\partial p}{\partial x} = -\rho\frac{\partial u}{\partial t} \qquad \frac{\partial u}{\partial x} = -\kappa\frac{\partial p}{\partial t}$$

Apply ∂/ ∂t on the right eqn: $\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial^2 u}{\partial x \partial t} = -\kappa\frac{\partial^2 p}{\partial t^2}$

Apply ∂/ ∂x on the left eqn: $\frac{\partial^2 p}{\partial x^2} = -\rho\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t}\right) \Rightarrow \frac{\partial^2 u}{\partial x \partial t} = -\frac{1}{\rho}\frac{\partial^2 p}{\partial x^2}$

Combining gives: $-\kappa\frac{\partial^2 p}{\partial t^2} = -\frac{1}{\rho}\frac{\partial^2 p}{\partial x^2}$

leading to $\frac{\partial^2 p}{\partial x^2} - \kappa\rho\frac{\partial^2 p}{\partial t^2} = 0$

The linear wave equation

Introduce the speed of sound:

$$c = \sqrt{\frac{1}{\rho\kappa}}$$

Finally gives the linear wave equation:

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

Solutions to the wave equation

Assume harmonic solution: $p(t,x) = p_0 \sin(\omega_0(t - \frac{x}{c}))$

Insert into the equation:

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

which shows this is a solution

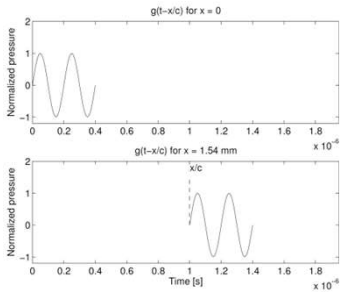
$$(-p_0\left(\frac{\omega_0}{c}\right)^2 + p_0\frac{1}{c^2}\omega_0^2)\sin(\omega_0(t - \frac{x}{c})) = 0$$

Note that c is speed of sound, which links spatial position and time.

General solutions: $p(t,x) = g(t \pm \frac{x}{c})$

$\frac{x}{c}$ is time delay

Link between speed of sound and delay



Propagation of 2-cycle 5 MHz pulse in a medium with a speed of sound of 1540 m/s

Table of speed of sound and densities

Medium	Density kg/m^3	Speed of sound m/s	Characteristic acoustic impedance kg/[m^2·s]
Air	1.2	333	0.4 $\times 10^3$
Blood	1.06 $\times 10^3$	1566	1.66 $\times 10^6$
Bone	1.38 – 1.81 $\times 10^3$	2070 – 5350	3.75 – 7.38 $\times 10^6$
Brain	1.03 $\times 10^3$	1505 – 1612	1.55 – 1.66 $\times 10^6$
Fat	0.92 $\times 10^3$	1446	1.33 $\times 10^6$
Kidney	1.04 $\times 10^3$	1567	1.62 $\times 10^6$
Lung	0.40 $\times 10^3$	650	0.26 $\times 10^6$
Liver	1.06 $\times 10^3$	1566	1.66 $\times 10^6$
Muscle	1.07 $\times 10^3$	1542 – 1626	1.65 – 1.74 $\times 10^6$
Spleen	1.06 $\times 10^3$	1566	1.66 $\times 10^6$
Distilled water	1.00 $\times 10^3$	1480	1.48 $\times 10^6$

Approximate densities, sound speeds, and characteristic acoustic impedances of humand tissues (data from the compilation by Goss et al. (1978) and (1980)). The speed of sound has been calculated from the density and characteristic impedance data.

Reflection and convolution

Receive signal for three events:

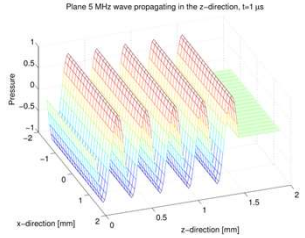
$$\begin{aligned} y(t) &= r_1(t_1)g(t-2t_1) \\ &+ r_2(t_2)g(t-2t_2) \\ &+ r_3(t_3)g(t-2t_3) \\ &+ \cdots \\ &= \sum_{i=1}^{N} r_i(t)g(t-2t_i) \end{aligned}$$

Convolution model:

$$y(t) = \int_{-\infty}^{+\infty} r(\theta)g(t-\theta)d\theta = r(t) * g(t)$$

Types of waves

Types of waves: Plane wave

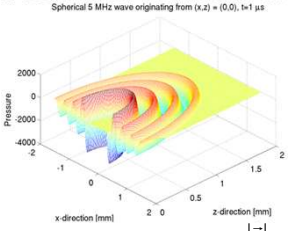


$$p(t,\vec{r}) = p_0 \sin(2\pi f_0 (t - \frac{z}{c}))$$

Propagates and oscillates in z-direction. Constant phase in x and y direction.

Used for intensity measures and description of reflection.

Types of waves: Spherical wave



$$p(t,\vec{r}) = \frac{p_0}{\vec{r}} \sin(2\pi f_0 (t - \frac{|\vec{r}|}{c}))$$

Propagates radially from a center point

Used for describing diffraction, focusing, and ultrasound fields

Both are mathematical abstractions used in describing ultrasound

Velocity of particle

Relation between pressure and particle velocity:

$$\frac{\partial p}{\partial x} = -\rho \frac{\partial u}{\partial t}$$

Finding the relationship between pressure and particle velocity for harmonic plane wave

$$p(t,x) = p_0 \sin(\omega_0 (t - \frac{x}{c}))$$

$$u(t) = -\frac{1}{\rho}\int \frac{\partial p}{\partial x} dt = +\frac{1}{\rho} p_0 \omega_0 \frac{1}{c}\int \cos(\omega_0 (t - \frac{x}{c}))dt$$

$$= \frac{1}{\rho c} p_0 \sin(\omega_0 (t - \frac{x}{c})) = \frac{p(t)}{\rho c} = p(t)/z$$

z = ρc - Characteristic acoustic impedance of medium

"Ohm's law" of acoustics:

$$p(t) = zu(t)$$

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Example

Typical values: c = 1500 m/s, ρ = 1000 kg/m^3
z = 1.5×106 kg/[m2s] = 1.5 MRayl
p_0 = 100 kPa, w_0 = 2π×5×10^6 rad/s

gives

$$u = \frac{100 \times 10^3}{1.5 \times 10^6} = 0.06 \text{ m/s}$$

Displacement:

$$\int u(t)dt = \frac{u}{\omega_0} = 2.1 \text{ nm}$$

Actual particle displacements are very, very small

Safety and Intensity of Ultrasound

Intensities

Defined as power per unit area [W/m^2]
For a plane wave:

$$I = \frac{1}{T}\int_0^T p(t) \cdot u(t)dt$$

For

$$p(t,x) = p_0 \sin(\omega_0(t - \frac{x}{c}))$$

$$u(t,x) = p(t,x)/z = \frac{p_0}{z}\sin(\omega_0(t - \frac{x}{c}))$$

this gives

$$I = \frac{p_0^2}{2z}$$

in [W/m^2] or more convenient in ultrasound [mW/cm^2]

Intensity measures

Instantaneous intensity:

$$I_i(t,\vec{r}) = \frac{p(t,\vec{r})^2}{z}$$

Spatial Peak Temporal Average intensity (I_{spta}):

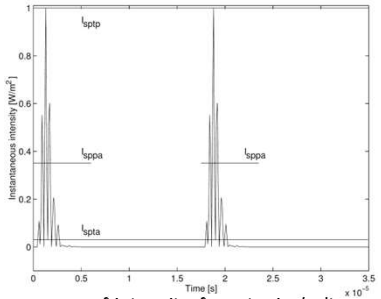
$$I_{spta}(t,\vec{r}) = \frac{1}{T}\int_0^T I_i(t,\vec{r})dt$$

Spatial Peak Temporal Peak intensity (I_{sptp}):

$$I_{sptp}(t,\vec{r}) = \max\{I_i(t,\vec{r})\}$$

I_{SPPA} – Spatial Peak Pulse Average intensity: mean over pulse duration

Example of intensities



Different measures of intensity for a typical ultrasound pulse.
The pulse repetition time is set artificially low at 17.5 µs

FDA safety limits

Use	$I_{spta.3}$ mW/cm² In Situ	$I_{spta.3}$ mW/cm² Water	$I_{sppa.3}$ mW/cm² In Situ	$I_{sppa.3}$ mW/cm² Water	$I_{m.3}$ mW/cm² In Situ	$I_{m.3}$ mW/cm² Water	MI
Cardiac	430	730	190	240	160	600	1.90
Peripheral vessel	720	1500	190	240	160	600	1.90
Ophthalmic	17	68	28	110	50	200	0.23
Fetal imaging	94	170	190	240	160	600	1.90

Highest known acoustic field emissions for commercial scanners as stated by the United States FDA (the use marked (a) also includes intensities for abdominal, intra-operative, pediatric, and small organ (breast, thyroid, testes, neonatal cephalic, and adult cephalic) scanning) from the September 9, 2008, FDA Guidance

Mechanical Index MI

Definition:

$$MI = \frac{P_{r.3}(z_{sp})}{\sqrt{f_c}}$$

Variables:

$P_{r.3}$ – Peak negative pressure in MPa derated by 0.3 dB/(MHz·cm)

z_{sp} – Point on the beam axis with the maximum pulse intensity integral

f_c – Center frequency in MHz

Example

Plane wave with an intensity of 730 mW/cm² (cardiac, water)
Pulse emitted: f0 = 5MHz, M = 2 periods, Tprf = 200 µs
Pressure emitted:

$$I_{spta} = \frac{1}{T_{prf}} \int_0^{M/f_0} I_i(t,r)dt = \frac{(M/f_0)}{T_{prf}} \frac{P_0^2}{2z}$$

$$p_0 = \sqrt{\frac{I_{spta} 2zT_{prf}}{M/f_0}} = 3.35 \times 10^6 \text{ Pa} = 33.5 \text{ atm}$$

(Rock concert at 120 dB (pain level) is 50 Pa)
The normal atmospheric pressure is 100 kPa

Example continued

The particle velocity is $U_0 = \frac{p_0}{Z} = \frac{3.35\times10^6}{1.48\times10^6} = 2.18$ m/s

The particle velocity is the derivative of the particle displacement, so

$$z(t) = \int U_0 \sin(\omega_0 t - kz)dt = \frac{U_0}{\omega_0}\cos(\omega_0 t - kz)$$

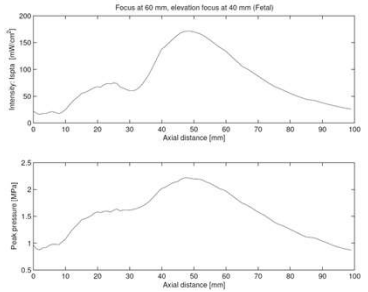
The displacement is $z_0 = \frac{U_0}{\omega_0} = 2.18/(2\pi\cdot5\cdot10^6) = 69$ nm

Mechanical Index MI is without deration

$$MI = \frac{P_{r.3}(z_{sp})}{\sqrt{fc}} = \frac{3.35}{\sqrt{5}} = 1.5$$

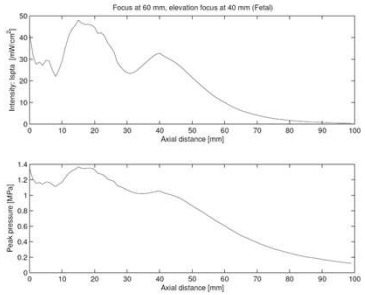
at 7 cm with deration it is $\frac{3.35\cdot10^{-0.3\times7\times5\times20}}{\sqrt{5}} = 0.44$

Typical intensities for fetal scanning in water



Simulated intensity profile in water for fetal scanning using a 65 elements 5 MHz linear array focused at 60 mm with an elevation focus at 40 mm

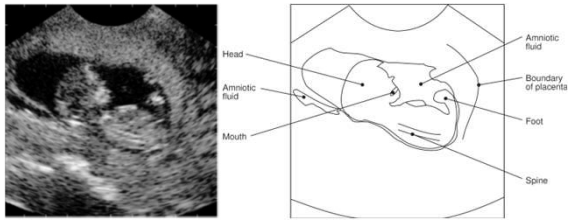
Typical intensities for fetal scanning in tissue



Simulated intensity profile in tissue for fetal scanning using a 65 elements 5 MHz linear array focused at 60 mm with an elevation focus at 40 mm

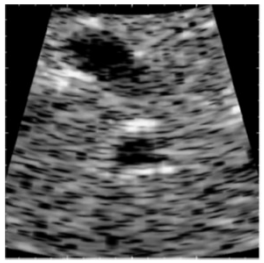
Scattering and imaging

Fetus – first trimester



Ultrasound image of 13th week fetus. The markers at the border of the image indicate one centimeter.

Speckle pattern in ultrasound image



4 x 4 cm image of a human liver for a 28-year-old male.
Speckle pattern in liver. The dark areas are blood vessels.

Backscattering

Power of backscattered signal:

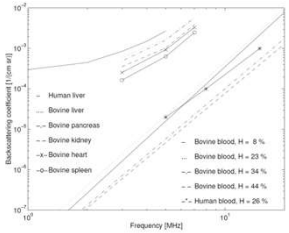
$$P_s = I_i \sigma_{sc}$$

I_i – Incident intensity [W/m^2]
σ_{sc} – Backscattering cross-section [m^2]
Received power of backscattered signal at transducer:

$$P_r = I_i \sigma_{sc} \frac{r^2}{4R^2}$$

r – Transducer radius
R – Distance to scattering region

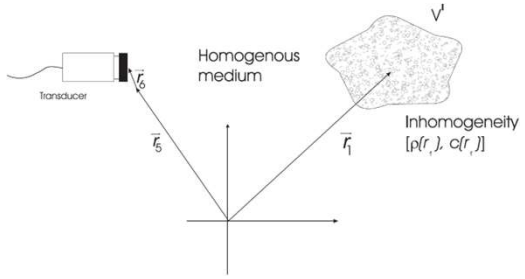
Backscattering coefficient



Measured backscattering coefficients for human liver tissue, bovine myocardium, bovine kidney cortex, human spleen, and bovine and human blood as a function of frequency

Backscattering coefficient – power received for a volume of scatterers.

Description of received signal



Theoretical model of received signal

$$p_r(\vec{r}_5,t) = v_{pe}(t) *_t \int_{V'} \left[\frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c_0} \right] h_{pe}(\vec{r}_1,\vec{r}_5,t)\, d^3\vec{r}_1$$

$$= v_{pe}(t) *_t h_{pe}(\vec{r}_1,t) *_r f_m(\vec{r}_1) = \mathrm{PSF}_{pe}(\vec{r}_1,t) *_r f_m(\vec{r}_1)$$

Electrical impulse response: $v_{pe}(t)$

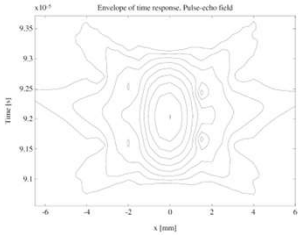
Transducer spatial response: $h_{pe}(\vec{r}_1,\vec{r}_5,t) = h_t(\vec{r}_1,\vec{r}_5,t) *_t h_r(\vec{r}_5,\vec{r}_1,t)$

Scattering term: $f_m(\vec{r}_1) = \frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c_0}$

Point spread function: $\mathrm{PSF}_{pe}(\vec{r}_1,t)$

$\vec{r}_1$	Position of transducer	$\vec{r}_5$	Position of scatterers
c	Speed of sound	ρ	Density of medium
$h(\vec{r}_5,\vec{r}_1,t)$	Spatial impulse response		
$\Delta\rho, \Delta c$	Perturbations in density and speed of sound		

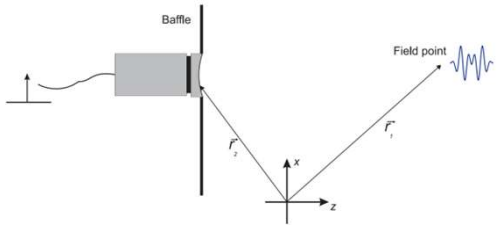
Point spread function



Basic model: $\mathrm{P}_t(\vec{r}_5,t) = \mathrm{PSF}_{pe}(\vec{r}_1,t) *_r f_m(\vec{r}_1)$

The topic of exercise 2 to investigate the influence of the point spread function

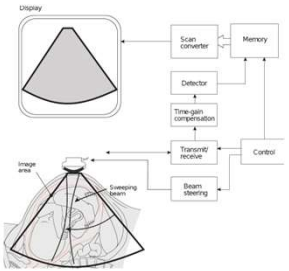
Linear acoustic system



Impulse response at a point in space (spatial impulse response).

We will return to this in the next lecture on ultrasound fields, focusing and imaging.

Signal processing in ultrasound system – Ex 1



Signal processing in a simple ultrasound system with a single element ultrasound transducer.

Signal processing

1. Acquire RF data (load from file)
2. Perform Hilbert transform to find envelope:

$$\mathrm{env}(n) = \left| y(n)^2 + H\{y(n)\}^2 \right|$$

3. Compress the dynamic range to 50 dB:

$$\log\_\mathrm{env}(n) = 20\log_{10}(\mathrm{env}(n))$$

4. Scale to the color map range (128 gray levels)
5. Make polar to rectangular mapping and interpolation
6. Display the image
7. Load a number of clinical images and make s movie

Polar to rectangular mapping and interpolation 1

Two step procedure to set-up tables and then perform interpolation

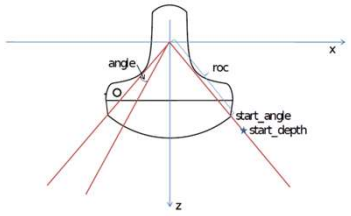
Table setup:

```
% Function for calculating the different weight tables for the
% interpolation. The tables are calculated according to the parameters in
% this function and are stored in the internal C-code.
%
% Input parameters:
%
% start_depth - Depth for start of image in meters
% image_size - Size of image in meters
%
% start_of_data - Depth for start of data in meters
% delta_r - Sampling internal for data in meters
% N_samples - Number of samples in one envelope line
%
% theta_start - Angle for first line in image
% delta_theta - Angle between individual lines
% N_lines - Number of acquired lines
```



```
%
% scaling - Scaling factor from envelope to image
% Nz - Size of image in pixels
% Nx - Size of image in pixels
%
% Output: Nothing, everything is stored in the C program
%
% Calling: make_tables (start_depth, image_size, ...
% start_of_depth, delta_r, N_samples, ...
% theta_start, delta_theta, N_lines, ...
% scaling, Nz, Nx);
% Version 1.1, 11/9-2001, JAJ: Help text corrected
```

Geometry



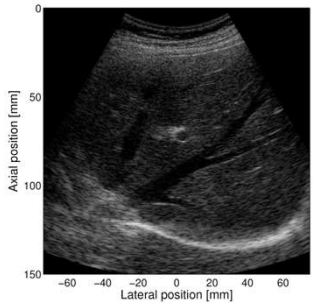
Relation between the different variables

Polar to rectangular mapping and interpolation 2

Perform the actual interpolation:

```
% Function for making the interpolation of an ultrasound image.
% The routine make_tables must have been called previously
%
% Input parameters:
%
% envelope_data - The envelope detected and log-compressed data as
% an integer array as 32 bits values (uint32)
%
% Output: img_data - The final image as 32 bit values
%
% Calling: img_data = make_interpolation(envelope_data);
```


Resulting image



Ultrasound image of portal veins in the liver.

Useful Matlab commands

Loading of files

```
load(['in_vivo_data/8820e_B_mode_invivo_frame_no_',num2str(j)]
```

Making a movie:

```
for j = 1:66
    image(randn(20))
    colormap(gray(256))
    axis image

    F(j) = getframes;
end

% Play the movie 5 times at 22 fr/s
movie(F, 5, 22)
```

Discussion for next time

Use the ultrasound system from the first discussion assignment

1. What is the largest pressure tolerated for cardiac imaging?
2. What displacement will this give rise to in tissue?

Prepare your Matlab code for exercise 1 for September 9