Danmarks Tekniske Universitet

Reconstruction in CT and hints to the assignments

Jørgen Arendt Jensen

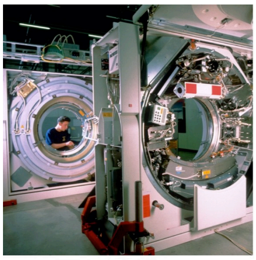
October 28, 2021

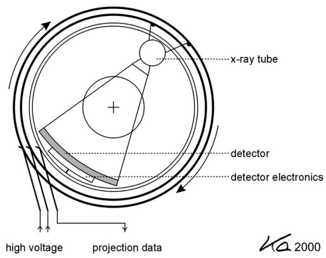
Center for Fast Ultrasound Imaging, Build 349
Department of Health Technology
Technical University of Denmark

1

CT reconstruction repetition & hints

- Filtered backprojection algorithm
  - Radon transform
  - Filtered backprojection
  - Filters and their impulse responses
- Advise for the assignments
- Reading material: Prince & Links Chapter 6

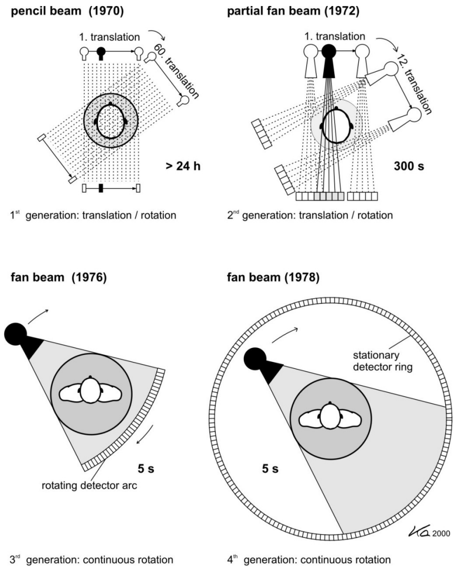




From: W. A. Kalender; Computed Tomography, Publicis, 2005

2

Modern CT system generations

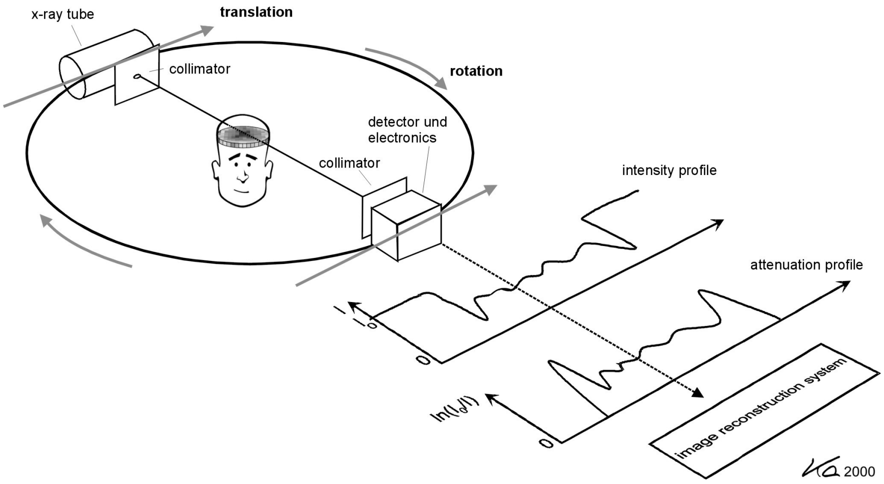


From: W. A. Kalender; Computed Tomography, Publicis, 2005

Center for Fast Ultrasound Imaging, Department of Health Technology
Technical University of Denmark

3

Measurement of attenuation

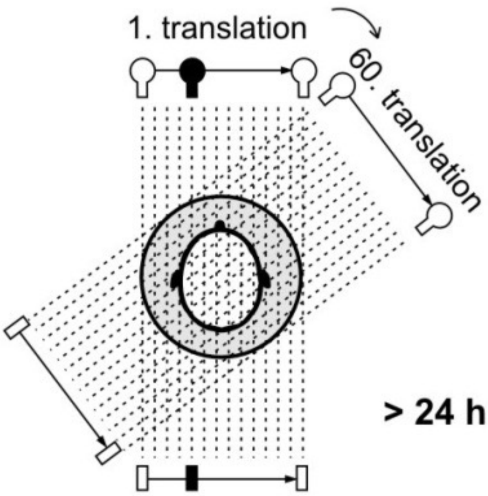


From: W. A. Kalender; Computer Tomography, Publicis, 2005

Center for Fast Ultrasound Imaging, Department of Health Technology
Technical University of Denmark

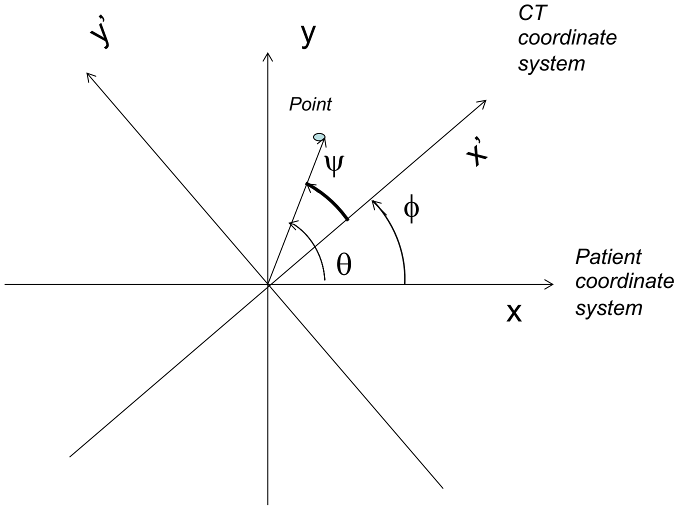
4

Parallel beam projection



From: W. A. Kalender; Computed Tomography, Publicis, 2005

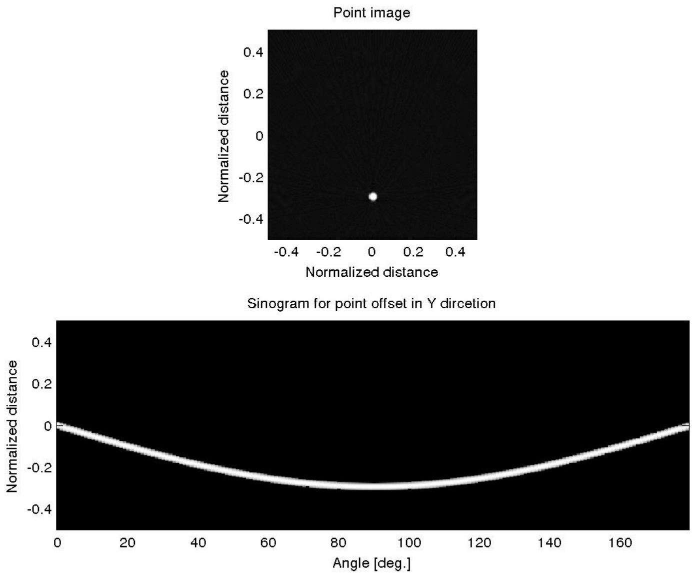
Parallel beam projection geometry

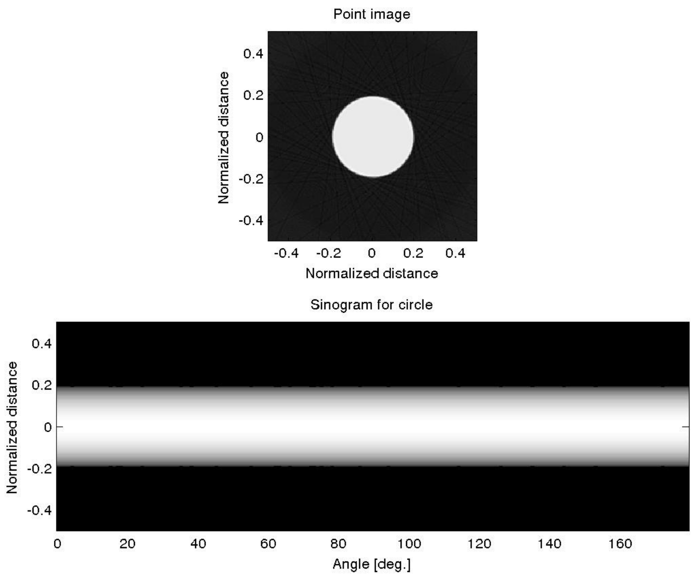


Radon transform

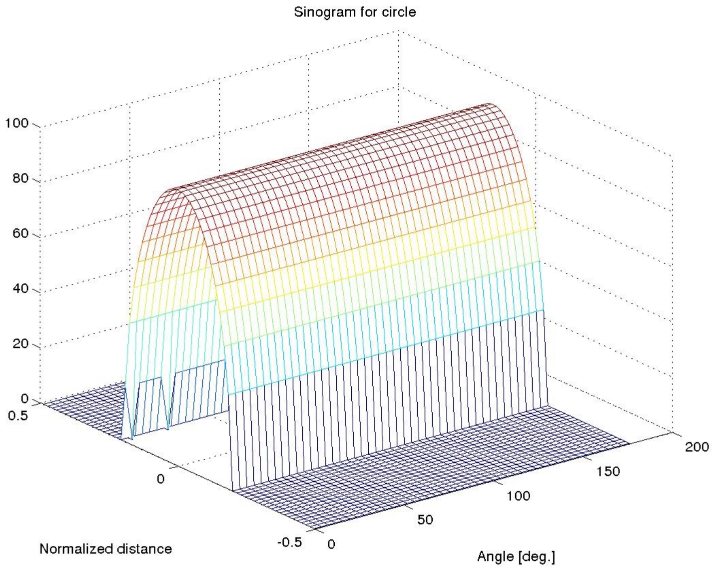
$$p(x',\phi) = \int_{-\infty}^{+\infty} f(x'\cos\phi - y'\sin\phi, x'\sin\phi + y'\cos\phi)dy'$$

f(x,y) – Attenuation image
x', y' – Gantry coordinate system
x, y – Patient coordinate system

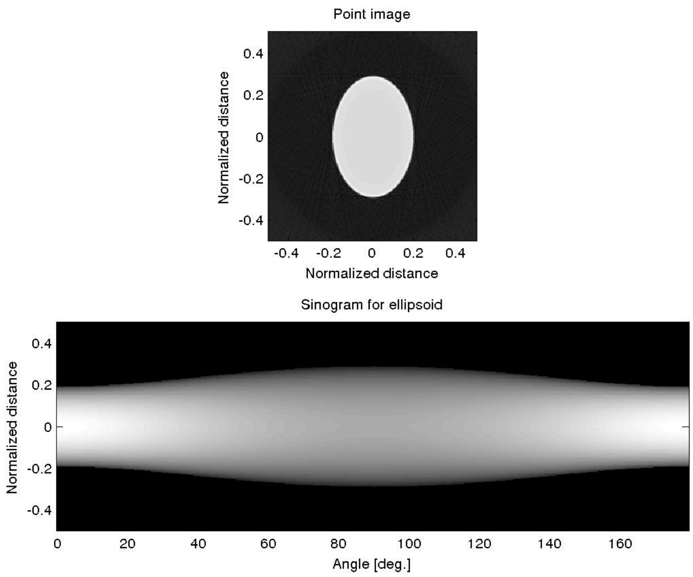




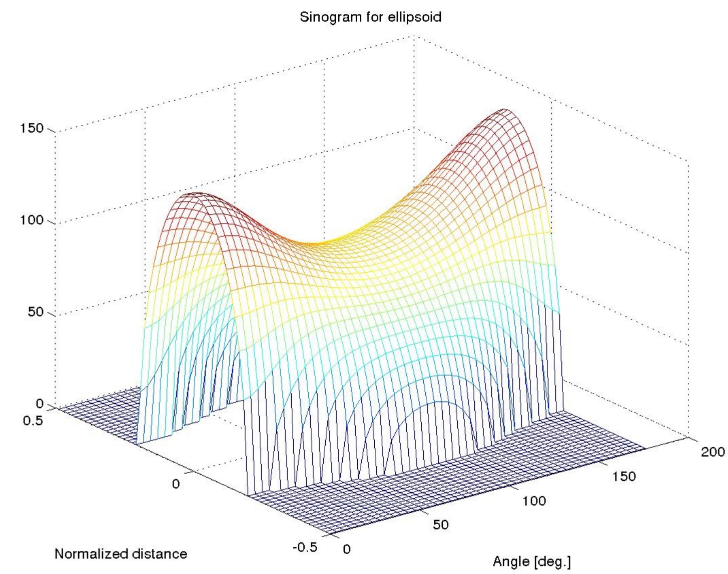
Point image
Normalized distance
0.4
0.2
0
-0.2
-0.4
-0.4 -0.2 0 0.2 0.4
Normalized distance
Sinogram for circle
Normalized distance
0.4
0.2
0
-0.2
-0.4
0 20 40 60 80 100 120 140 160
Angle [deg.]



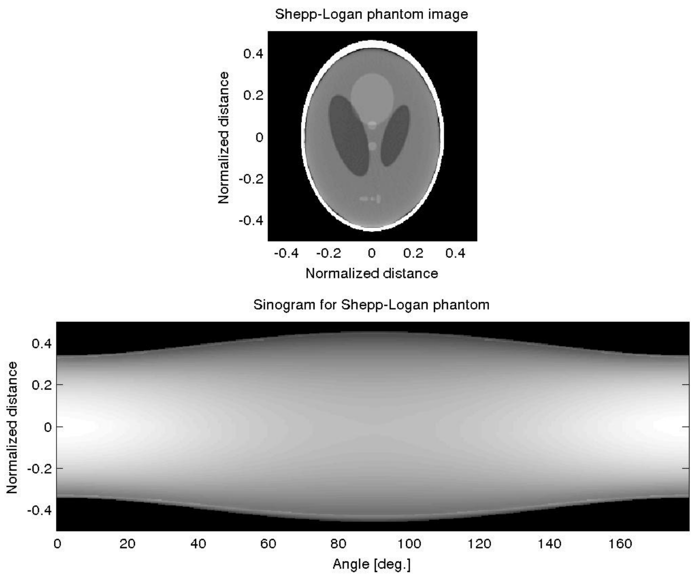
Sinogram for circle
100
80
60
40
20
0
0.5
0
-0.5
Normalized distance
0
50
100
150
200
Angle [deg.]


Point image
Normalized distance
0.4
0.2
0
-0.2
-0.4
-0.4 -0.2 0 0.2 0.4
Normalized distance
Sinogram for ellipsoid
Normalized distance
0.4
0.2
0
-0.2
-0.4
0 20 40 60 80 100 120 140 160
Angle [deg.]

11

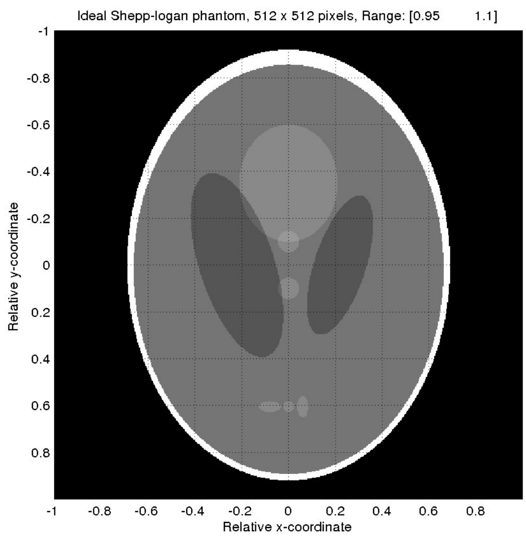

Sinogram for ellipsoid
150
100
50
0
0.5
0
-0.5
Normalized distance
0 50 100 150 200
Angle [deg.]

12



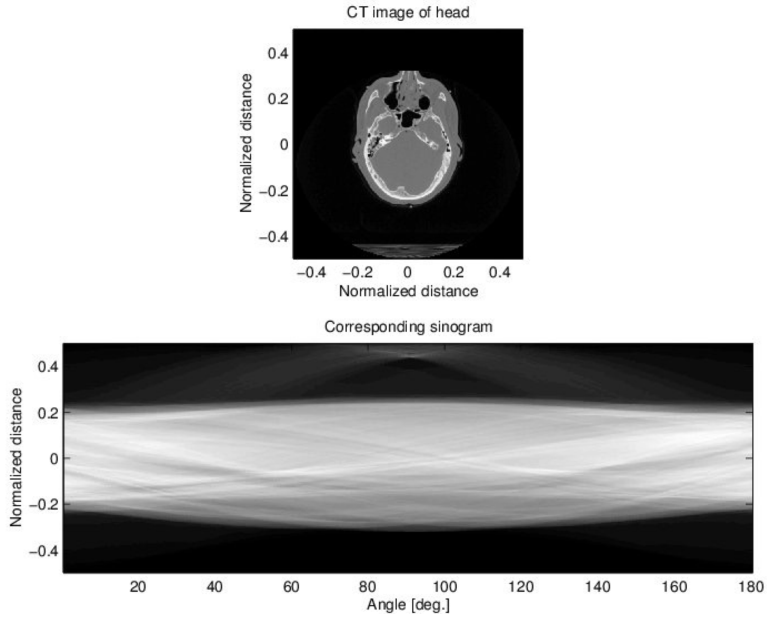
Center for Fast Ultrasound Imaging, Department of Health Technology
Technical University of Denmark

Shepp-Logan phantom



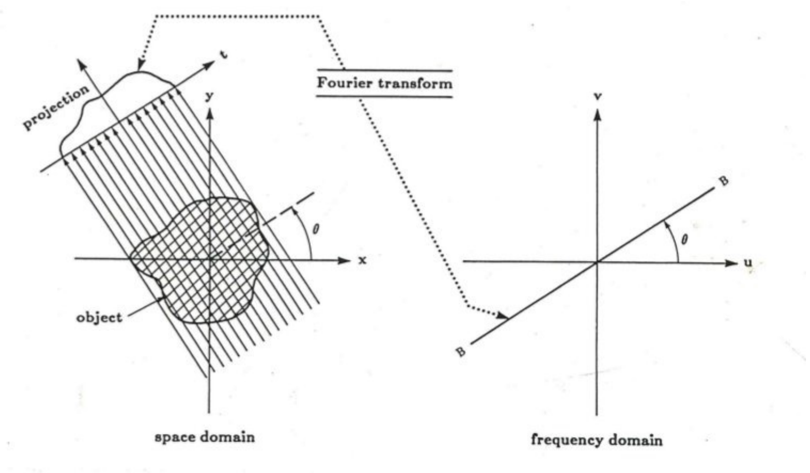
Note that the Shepp-Logan phantom is not in Hounsfield units, but the relative scaling is the same.

Center for Fast Ultrasound Imaging, Department of Health Technology
Technical University of Denmark



Demo in: for_13/matlab_demo/proj_demo

Fourier slice theorem



Demo in: for_13/matlab_demo/ct_demo

Fourier Slice Theorem

$$P(\rho,\phi) = \int_{-\infty}^{+\infty} p(x',\phi) e^{-j2\pi\rho x'} dx'$$

$$= F(\rho\cos\phi, \rho\sin\phi)$$

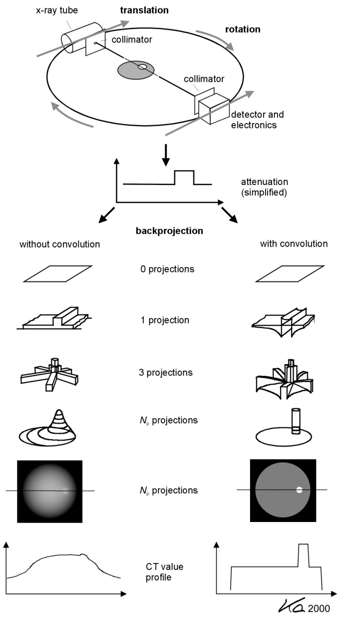
F(u,v) – 2D Fourier transform of image
ϕ – Gantry rotation

Filtered Back Projection (FBP)

$$\hat{f}(\mathrm{x},\mathrm{y}) = \int_{0}^{\pi} \int_{-\infty}^{+\infty} |\rho| P(\rho,\phi) e^{j2\pi\rho x'} d\rho d\phi$$

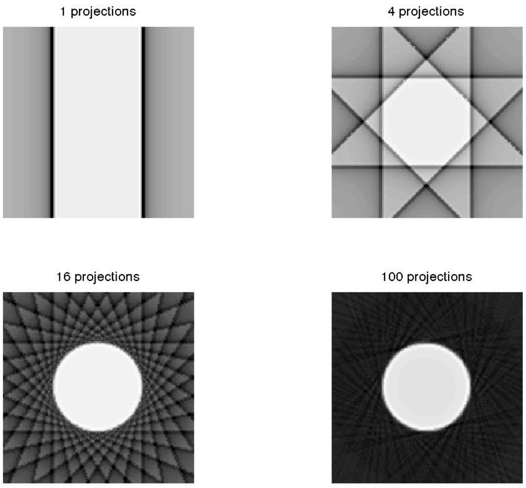
$\hat{f}(\mathrm{x},\mathrm{y})$ – Reconstructed image
ϕ – Gantry rotation
x' – Detector position

Filtered backprojection



- Perform for all projections:
  - Make Fourier transform of projected data
  - Apply filter in Fourier domain
  - Make invers transform
  - Back-project and sum with previous image

Influence from number of projections



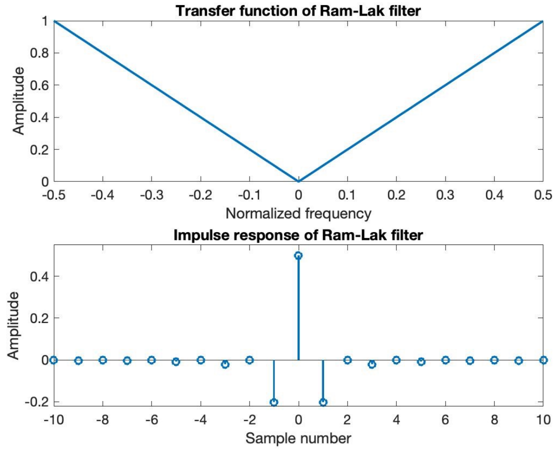
Ram-Lak filter

Transfer function:

$$h(\rho) = \begin{cases} |\rho|, & |\rho| \leq B \\ 0 & else \end{cases}$$

Impulse response

$$h(k) = \begin{cases} B^2 & k = 0 \\ -\dfrac{B^2}{\left(\dfrac{\pi}{2}k\right)^2} & k\ odd \\ 0 & k\ even \end{cases}$$



Impulse response for Ram-Lak filter

- Impulse response of square wave:

$$h_{square}(x) = A2B\,\frac{\sin 2\pi B\,x}{2\pi Bx}$$

- Ram-Lak impulse response: square wave – triangle:

$$h_{ramlak}(x) = 2BB\,\frac{\sin 2\pi Bx}{2\pi Bx} - B^2\,\frac{\sin^2 \pi Bx}{(\pi Bx)^2}$$

Sampling of impulse response for Ram-Lak filter

- Sampling at $\frac{1}{\Delta x} = 2B, x = k\ \Delta x = \frac{k}{2B}$:

$$h_{ramlak}(k) = 2B^2 \frac{\sin 2\pi B\frac{k}{2B}}{2\pi B\frac{k}{2B}} - B^2 \frac{\sin^2 \pi B\frac{k}{2B}}{(\pi B\frac{k}{2B})^2}$$

$$\sin 2\pi B\frac{k}{2B} = \sin \pi k = 0$$

$$\sin^2 \pi B\frac{k}{2B} = \sin^2 \frac{\pi}{2} k = 1\ (k\ odd) else\ 0$$

- Gives:

$$h(0) = B^2$$

$$h(k) = -B^2 \frac{\sin^2 k\frac{\pi}{2}}{(k\frac{\pi}{2})^2} = -\frac{B^2}{(k\frac{\pi}{2})^2} \text{ (k odd)}$$

$$h(k) = 0 \text{ else}$$

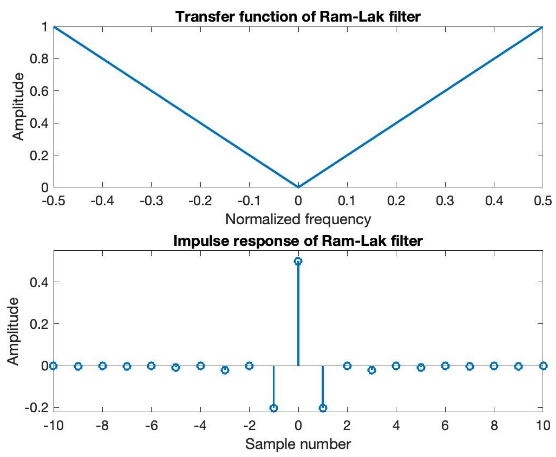
Ram-Lak filter

Transfer function:

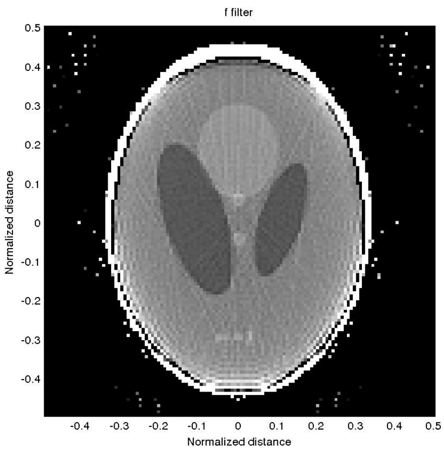
$$h(\rho) = \begin{cases} |\rho|, |\rho| \le B \\ 0 \quad else \end{cases}$$

Impulse response

$$h(k) = \begin{cases} B^2 & k = 0 \\ -\frac{B^2}{\left(\frac{\pi}{2}k\right)^2} & k\ odd \\ 0 & k\ even \end{cases}$$



Transfer function of filters – Ram-Lak



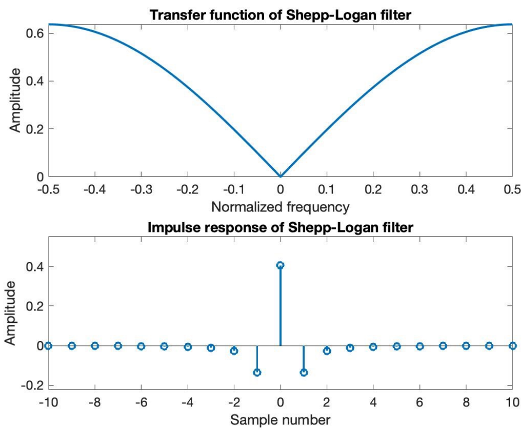
Shepp-Logan filter

Transfer function:

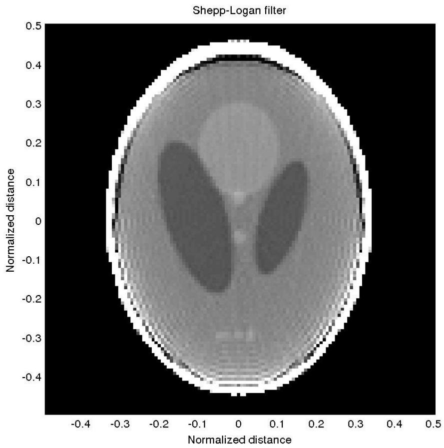
$$h(\rho) = \begin{cases} |\rho| \dfrac{sin\frac{2\pi\rho}{4B}}{\frac{2\pi\rho}{4B}}, |\rho| \leq B \\ 0 \quad else \end{cases}$$

Impulse response

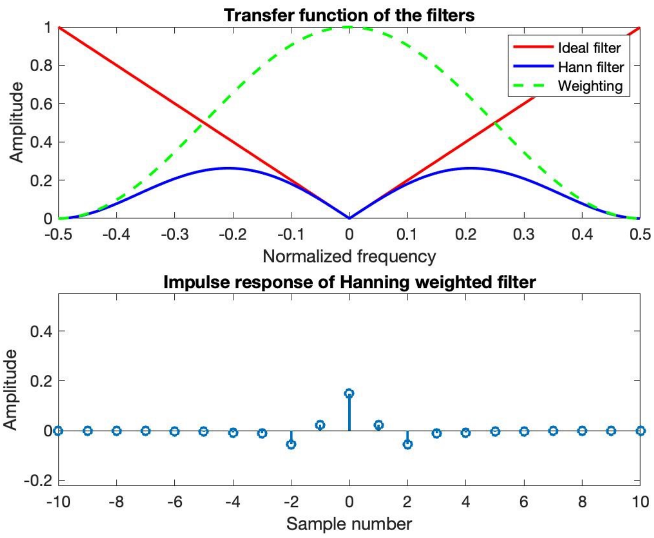
$$h(k) = -\frac{8B^2}{\pi(4k^2-1)}$$



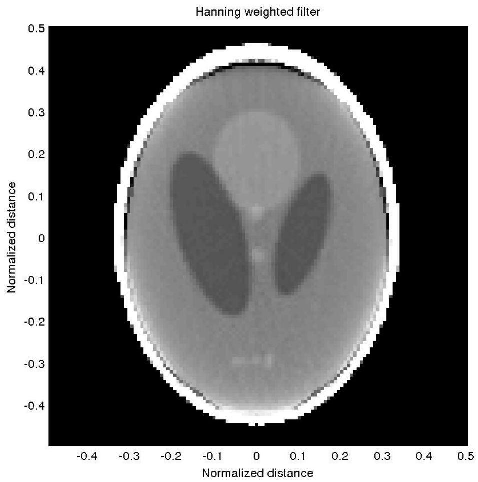
Shepp-Logan filter



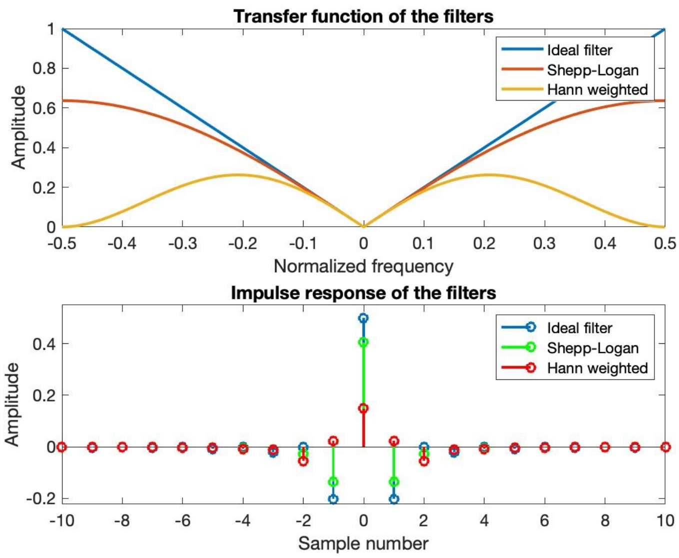
Hanning weighted filter



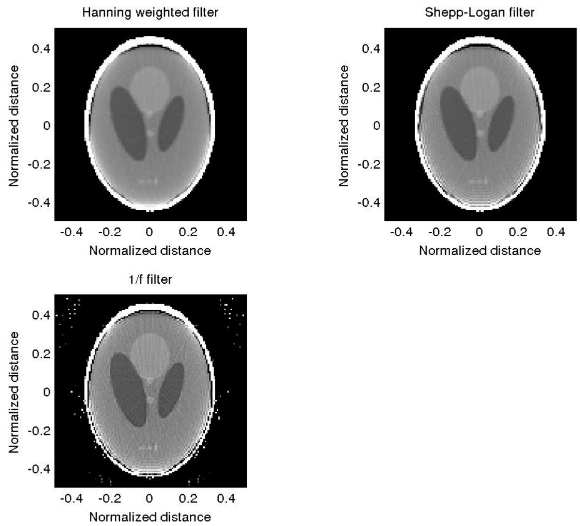
Hanning weighted filter



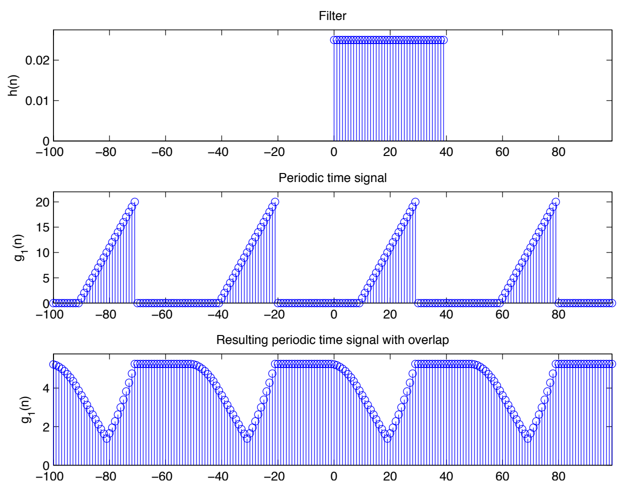
Filter transfer functions and impulse responses



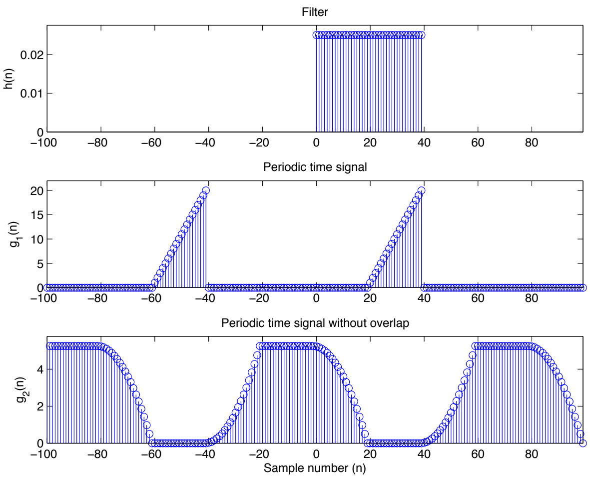
Comparison between filters



Circular convolution



Circular convolution

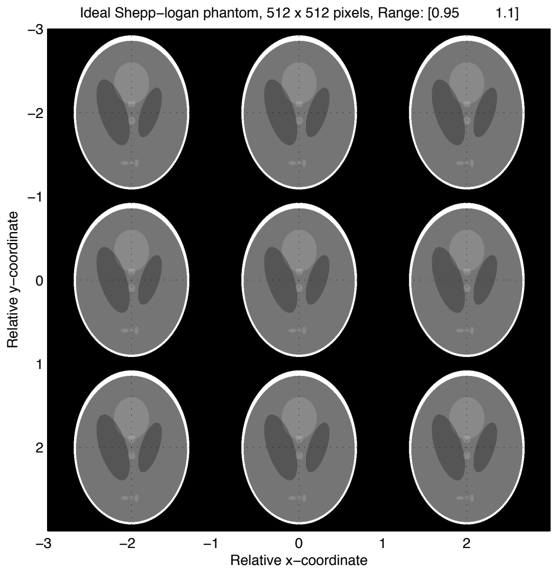


Center for Fast Ultrasound Imaging, Department of Health Technology
Technical University of Denmark

33/x

33

Circular convolution – Shepp-Logan

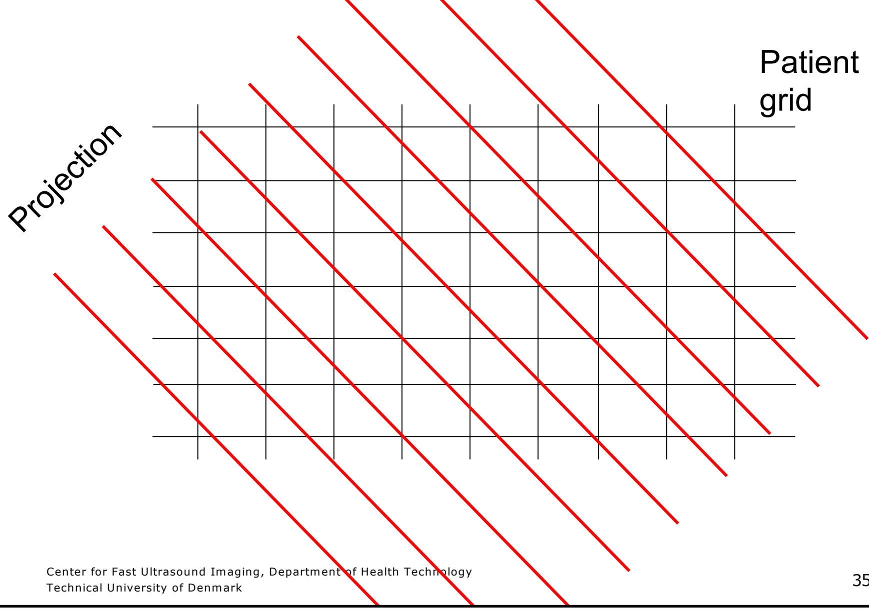


Center for Fast Ultrasound Imaging, Department of Health Technology
Technical University of Denmark

34/x

34

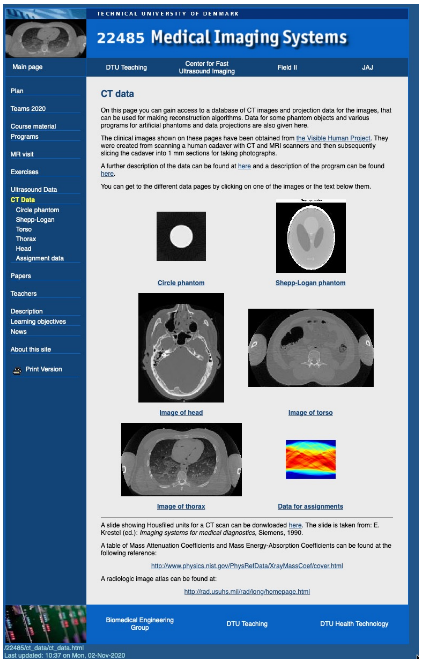
Backprojection



35

Data for testing and validation

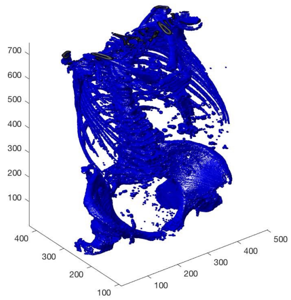
- Use data sets on web site
- Circular phantom for geometry test
- Shepp-Logan for orientation and quantitative data
- In-vivo images for Hounsfield units

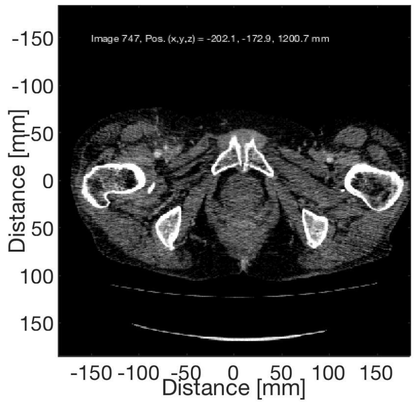


36

Assignment data

- DICOM data from female patient
- All data available on the web
- Task is to find which slice it is





Data and program in: undervisning/k_22485_31545_billeder/ct_data/dicom_data

Reading DICOM data

```
%  Set overall parameters
dir_name='DICOM/ST00001/SE00001/';  %  Directory name
start_image=1;                      %  First image in series
end_image=747;                      %  Last image in series
frame_rate=50;                      %  Frame rate for playing back the movie

%  Set the dynamic range for the display
off_set=100;          %  Offset [Hu]
range=400;            %  Range to display [Hu]
map_values=128;       %  Number of gray level values
bone_off_set= -250;   %  Offset for showing the bones
bone_range=100;       %  Range for showing the bones

%  Initialize figure
colormap(gray(map_values));
dicom_movie(end_image+1-start_image) = struct('cdata',[],'colormap',[]);

%  Read information for the first images
file_name='IM00001';
info=dicominfo([dir_name, file_name]);
dx=info.PixelSpacing(1);
dy=info.PixelSpacing(2);
Y = dicomread(info);
[Nx,Ny]=size(Y);

%  Make space for all the images
Y=zeros(Nx,Ny,end_image+1-start_image);
z_positions=zeros(end_image+1-start_image,1);
```

```
%  Loop through the images and read and display them

for i=start_image:end_image
    file_name=['IM00',num2str(floor(i/100)),num2str(floor(rem(i,100)/10)),num2str(rem(i,10))];
    info=dicominfo([dir_name, file_name]);
    Y(:,:,i) = dicomread(info);
    image(((1:Nx)-Nx/2)*dx,((1:Ny)-Ny/2)*dy,(double(Y(:,:,i))+off_set)/range*map_values)
    xlabel('Distance [mm]')
    ylabel('Distance [mm]')
    pos=sprintf('%5.1f, %5.1f, %5.1f', info.ImagePositionPatient(1), …
         info.ImagePositionPatient(2), info.ImagePositionPatient(3));
    z_positions(i)= info.ImagePositionPatient(3);
    text(-150, -150, ['Image ', num2str(i),', Pos. (x,y,z) = ', pos,' mm'], 'Color', [1 1 1])
    axis('image')
    drawnow
    dicom_movie(i)=getframe;
end

%  Display the movie

movie(dicom_movie, 5, frame_rate);
```

Full script can be found at:

courses.healthtech.dtu.dk/22485/files/ct_data/dicom_data/display_dicom_images.m

on the page for the CT data: courses.healthtech.dtu.dk/22485/?ct_data/assign_data.html