Directional regularization for the limited-angle Helsinki Tomography Challenge with the Core Imaging Library (CIL)

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Joint work with

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Evangelos Papoutsellis – UKRI STFC, Finden

Open challenge on limited-angle CT reconstruction

HTC 2022

HELSINKI TOMOGRAPHY CHALLENGE

https://www.fips.fi/HTC2022.php

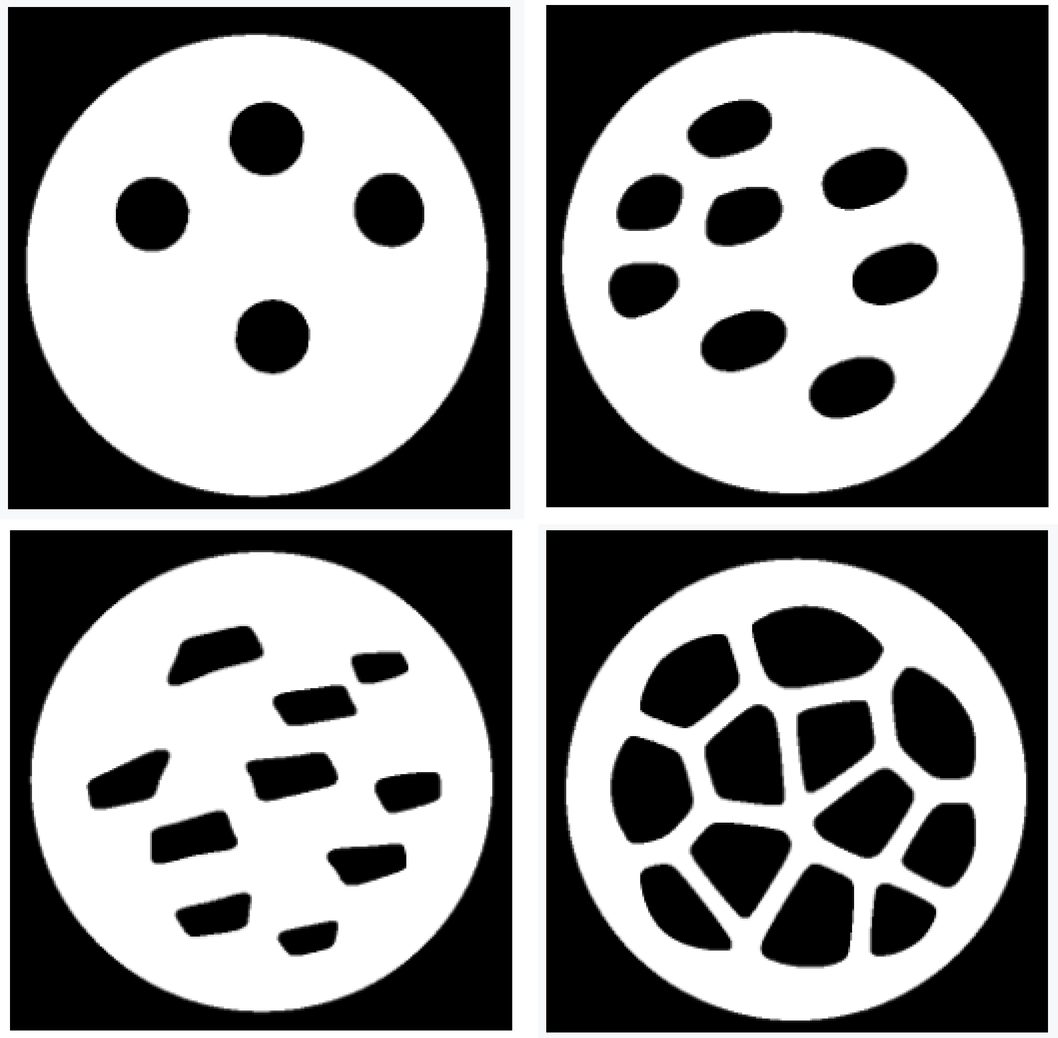
CIL team entry finished 3rd

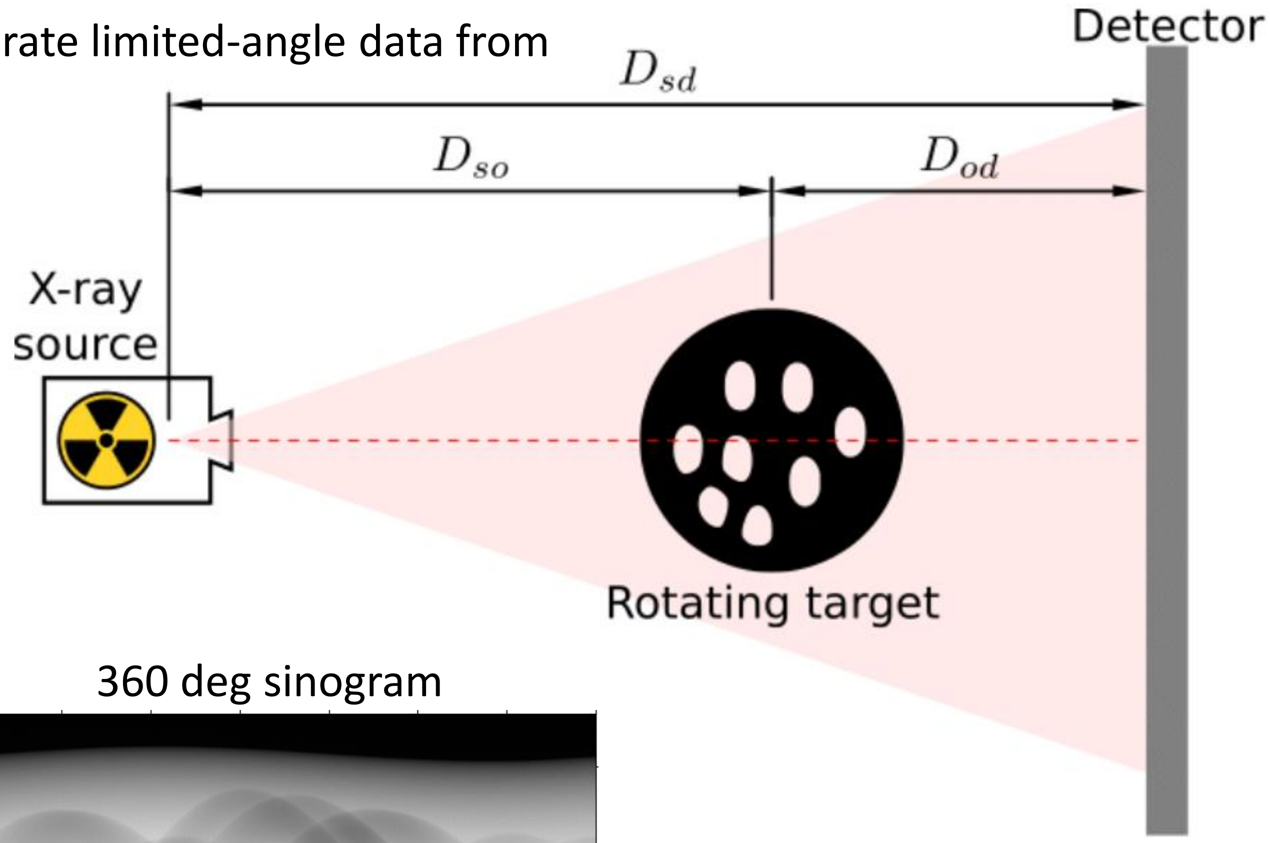
– beaten by two teams using machine learning
with large amounts of synthetically generated training data

CIL methods described in preprint:
https://arxiv.org/abs/2310.01671

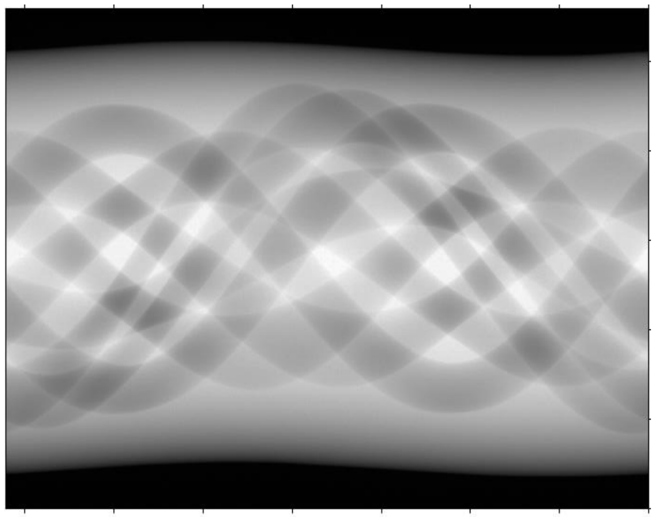
Test data provided and scan setup

- Four 360 degree measured sinogram data sets to generate limited-angle data from
- Reference reconstructions and segmentations





360 deg sinogram



Challenge levels

Table 1: Limited-angle tomography difficulty groups

Group	Angular range	Angular increment	Number of projections
1	90°	0.5°	181
2	80°	0.5°	161
3	70°	0.5°	141
4	60°	0.5°	121
5	50°	0.5°	101
6	40°	0.5°	81
7	30°	0.5°	61

Assessment of (segmented) reconstructions

The reconstructions will be assessed quantitatively, comparing the reconstructed binary image I_r with the ground truth binary image I_t, assigning a numeric score. I_r is assumed to have a dimension of 512 x 512 pixels, otherwise a score 0 will be given to the reconstruction I_r.

The score is based on the confusion matrix of the classification of the pixels between empty (0) or material (1). The confusion matrix is composed by

$$TP = \sum_{i,j} (I_t \cap I_r)_{ij}$$

$$FP = \sum_{i,j} (\bar{I}_t \cap I_r)_{ij}$$

$$FN = \sum_{i,j} (I_t \cap \bar{I}_r)_{ij}$$

$$TN = \sum_{i,j} (\bar{I}_t \cap \bar{I}_r)_{ij}$$

$$\mathbf{M} = \begin{bmatrix} TP & FN \\ FP & TN \end{bmatrix}$$

The score of the reconstruction is given by the Matthews correlation coefficient (MCC)

$$S = \frac{TP \times TN - FP \times FN}{\sqrt{(TP + FP)(TP + FN)(TN + FP)(TN + FN)}},$$

where $S \in [-1, 1]$. A score of $+1$ (best) represents a perfect reconstruction, 0 no better than random reconstruction, and -1 (worst) indicates total disagreement between reconstruction and ground truth. A python code that implements the scoring will be provided to the competitors. The same code will be used to assess the algorithms.

The Grand Prize of HTC2022

On top of the unlimited glory, the winner also receives the **Ultimate Limited Angle Device**. It is a vintage-looking tool for everyday use where determining the angle (limited or not) is necessary.

The top participants of the challenge will be invited to a **minisymposium at the Inverse Days Conference** organized by the Finnish Inverse Problems Society (FIPS) to be held in Kuopio, Finland, in December 2022.

Special session at Inverse Days, Kuopio 2022


HTC 2022
HELSINKI TOMOGRAPHY CHALLENGE

Helsinki Tomography Challenge 2022 – Finnish Inverse Problems Society
Certificate of Award

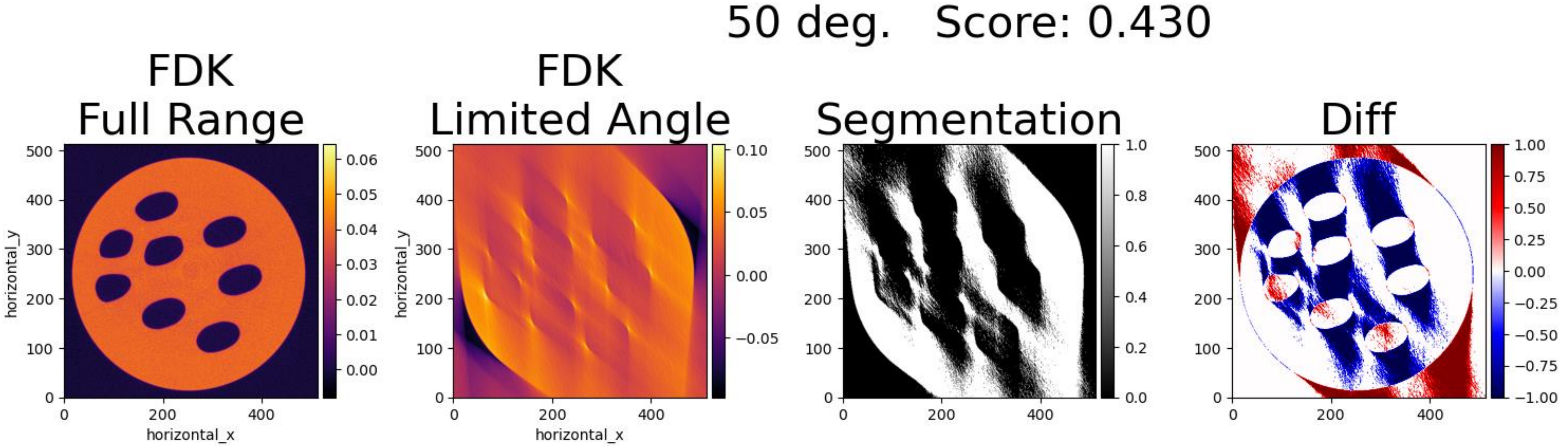
Presented to Jakob Sauer Jørgensen, Edoardo Pasca, Gemma Fardell, Evangelos Papoutsellis, and Laura Murgatroyd for participating as a team in the Helsinki Tomography Challenge 2022 held at the Department of Mathematics and Statistics of the University of Helsinki and getting the third place.

Fernando Moura
Organizing committee
Helsinki, November 24th 2022

fips Finnish Inverse Problems Society

UNIVERSITY OF HELSINKI
Department of Mathematics and Statistics

Motivation



- See how far "conventional" CT pre-processing plus variational methods could go
- Use existing general purpose CIL tools as much as possible – limited time for new dev

5 submissions, variations of

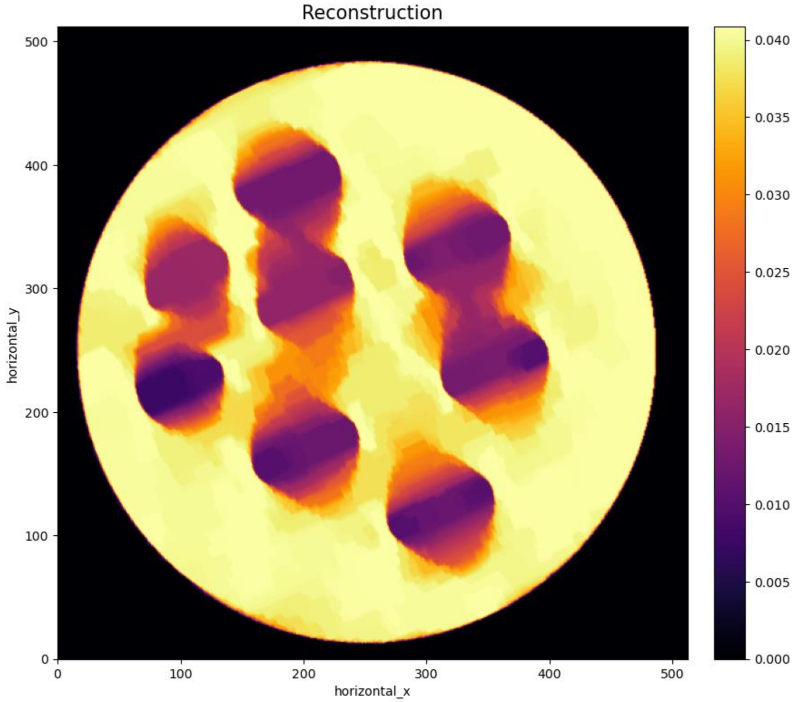
- Preprocessing
- Reconstruction
- Segmentation

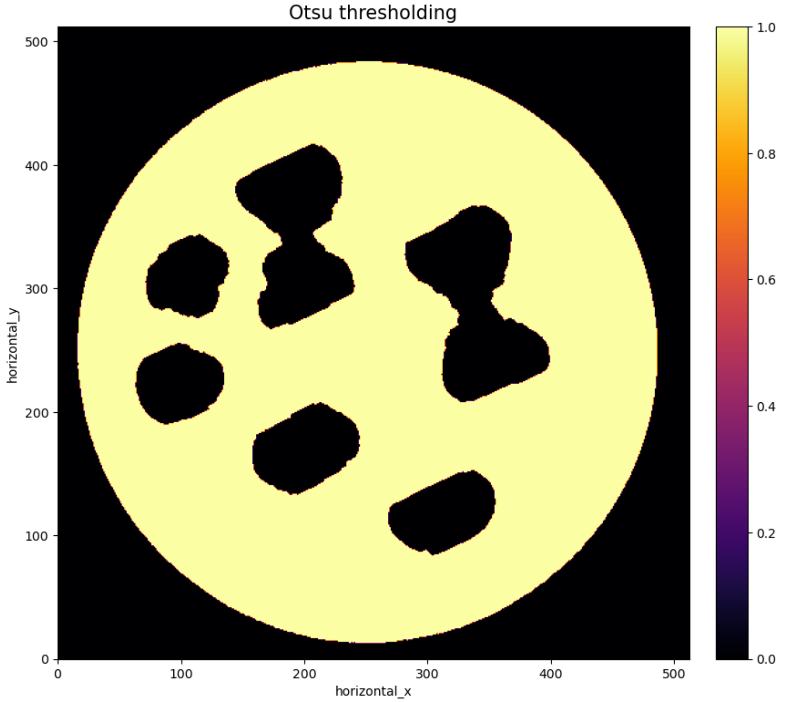
Segmentation

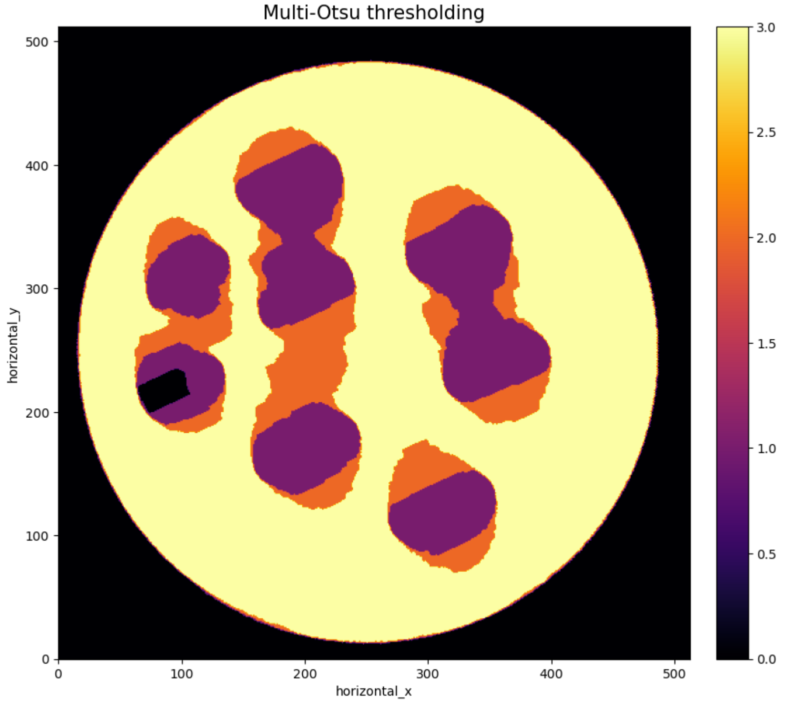
- 'blind' segmentation of the test data
- Not our expertise!
- Otsu triple-threshold worked consistently for the test data at 30 degrees

- Otsu thresholded segmentation
  - Identifies signal peak

- Otsu triple-thresholded segmentation
  - Strong signal
  - Messy signal
  - Messy background
  - Strong background

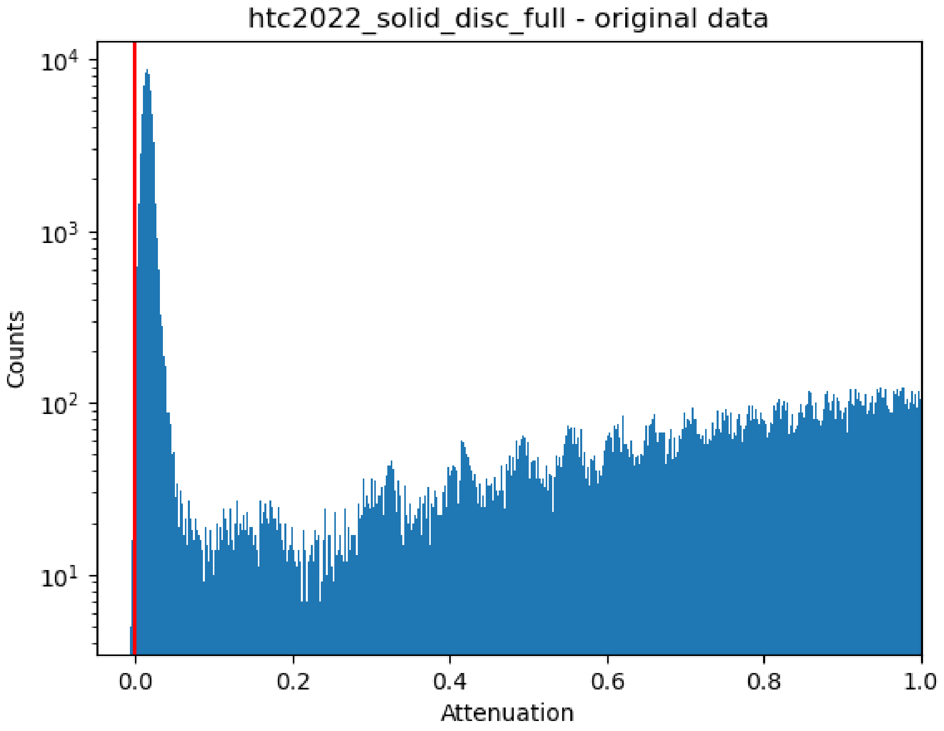


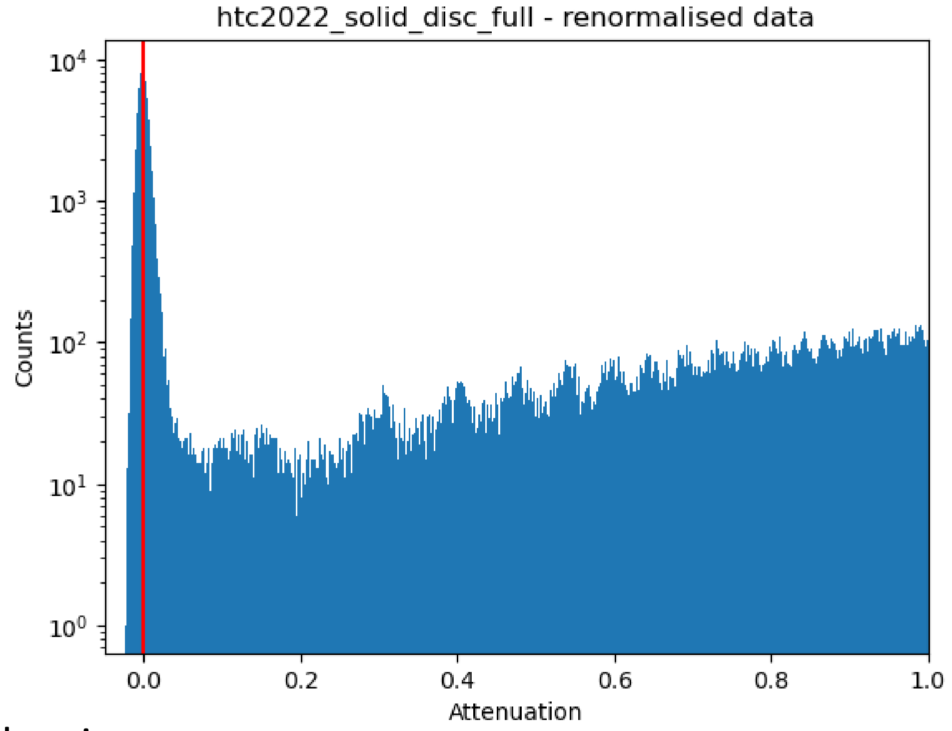




Renormalization of sinogram

- Background attenuation should have a mean at zero
- Test data had an offset i.e. the normalization image was brighter than the data
  - Convert data back to I/I0, renormalize for a peak at 1, convert back to absorption

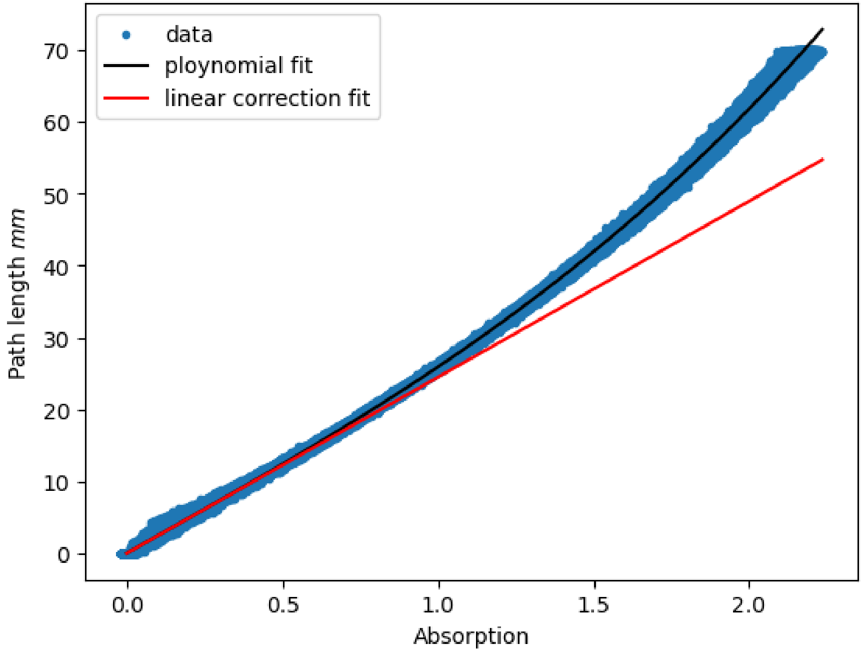


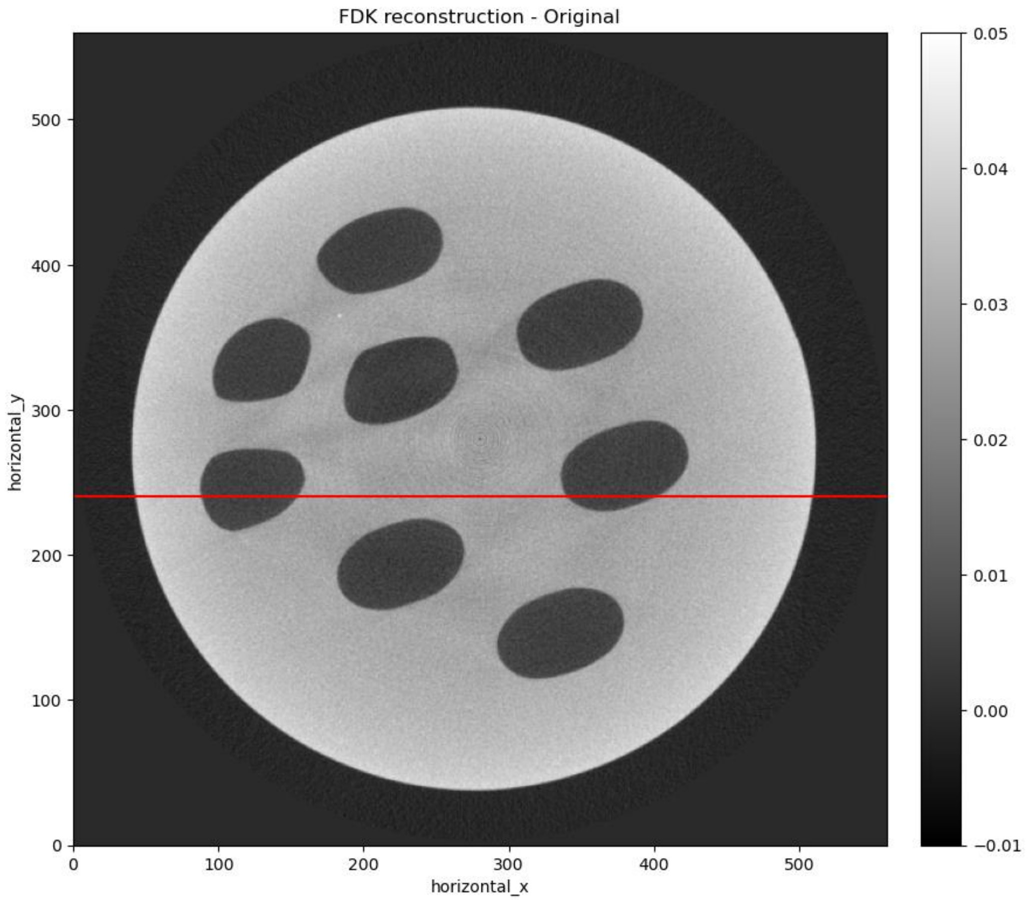


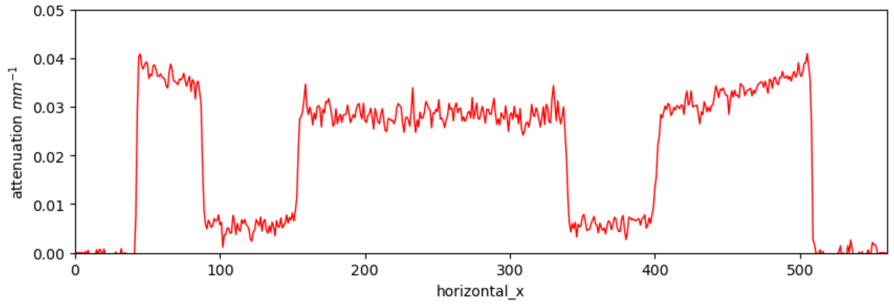
Histograms of the sinogram

Beam hardening correction

- Lower energy rays are preferentially absorbed leading to a non-linear measurement
- Single material scan can be linearised to an effective monochromatic energy
- Correction to the linear attenuation of acrylic at 24.7 KeV, mu = 0.0409 mm^{-1}







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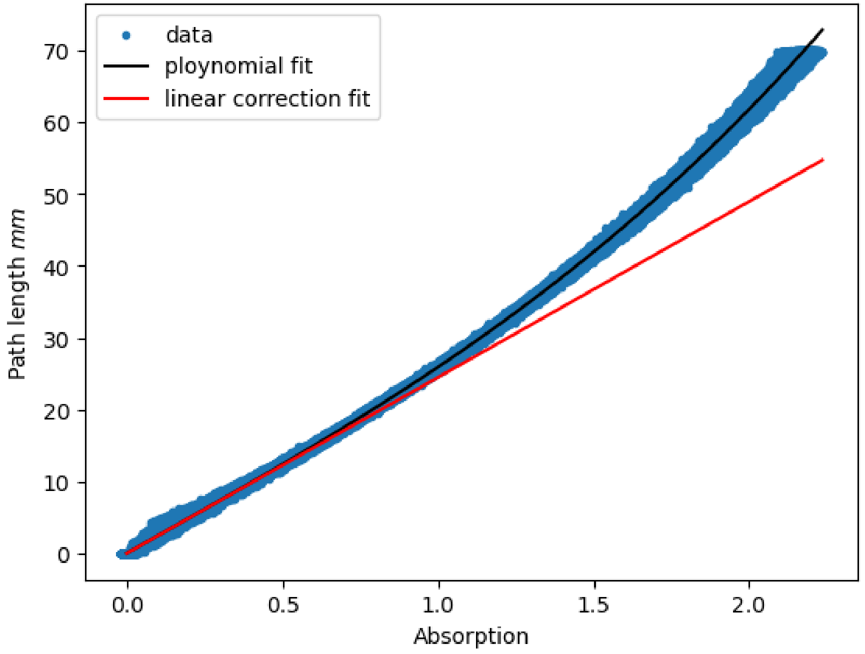
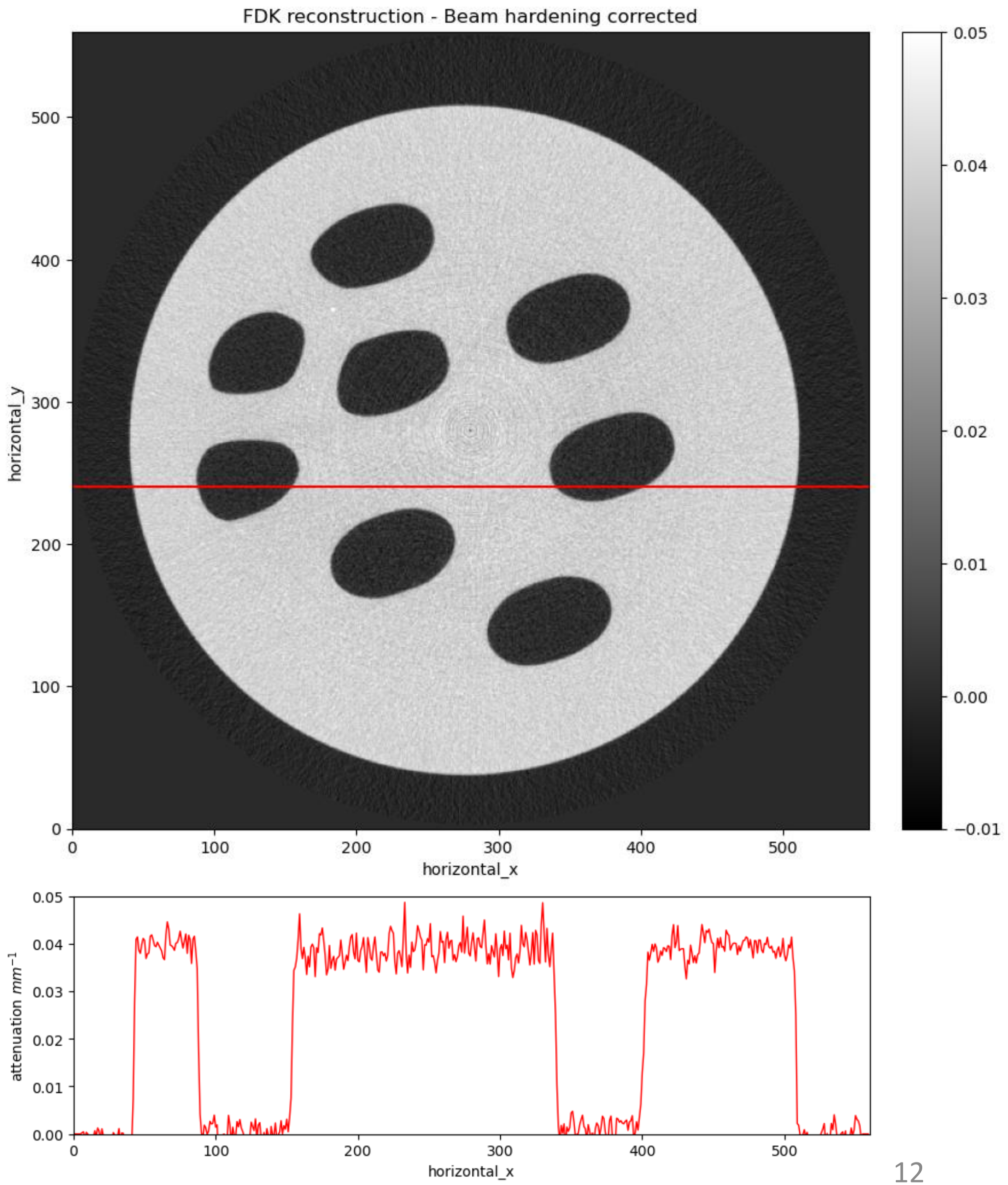
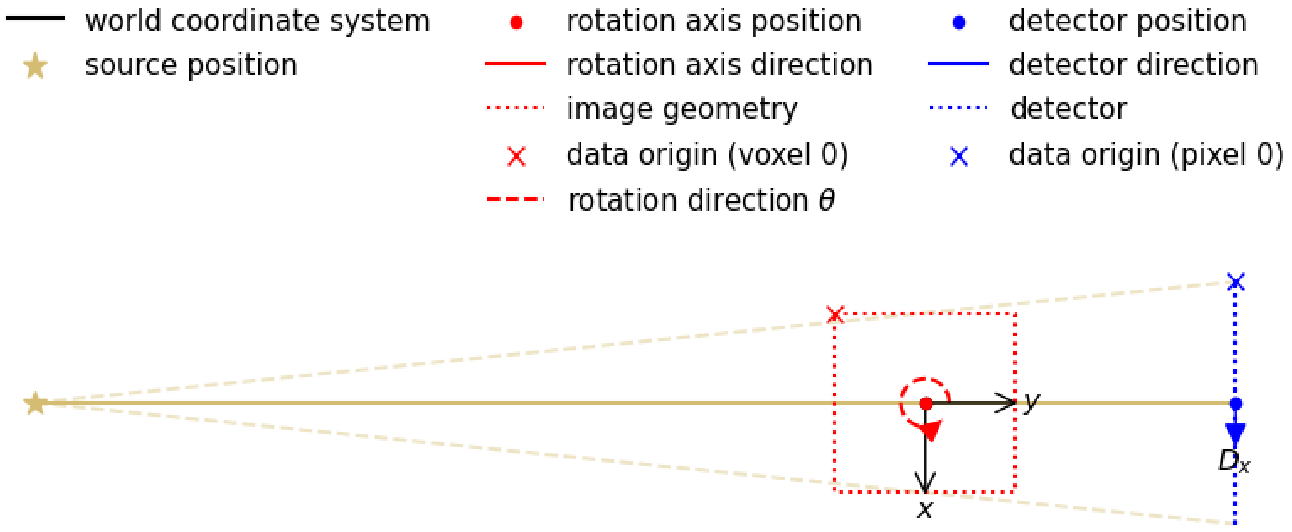
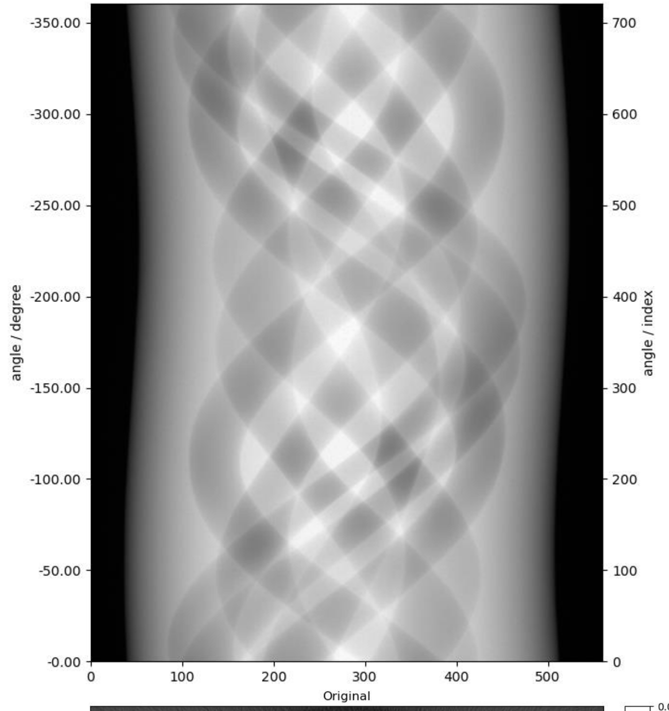


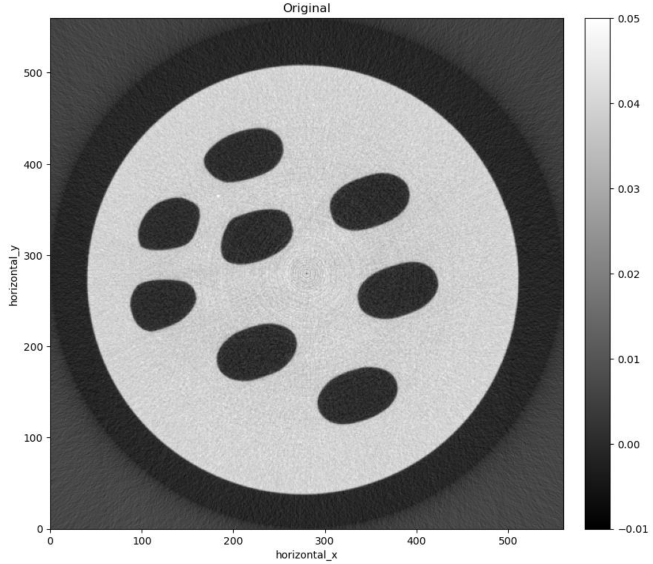


Zero-padding

- The reconstruction window extends outside the field of view
- Causes a non-zero background outside the radius of the detector
- Zero-Padding the acquisition data corrects this







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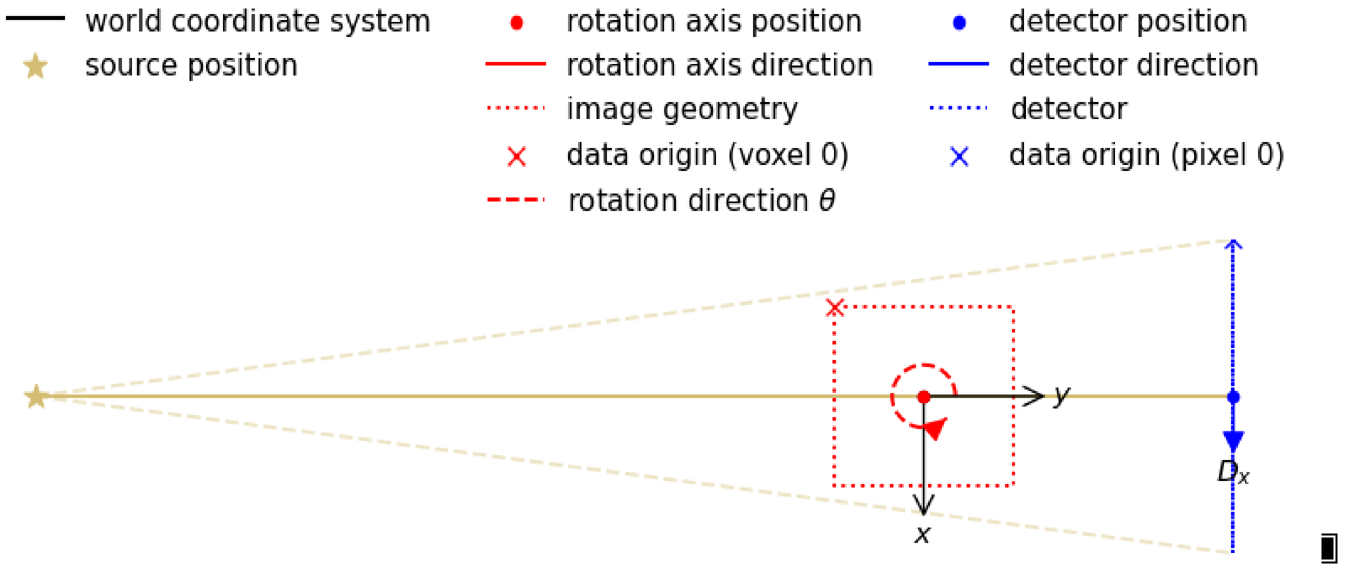
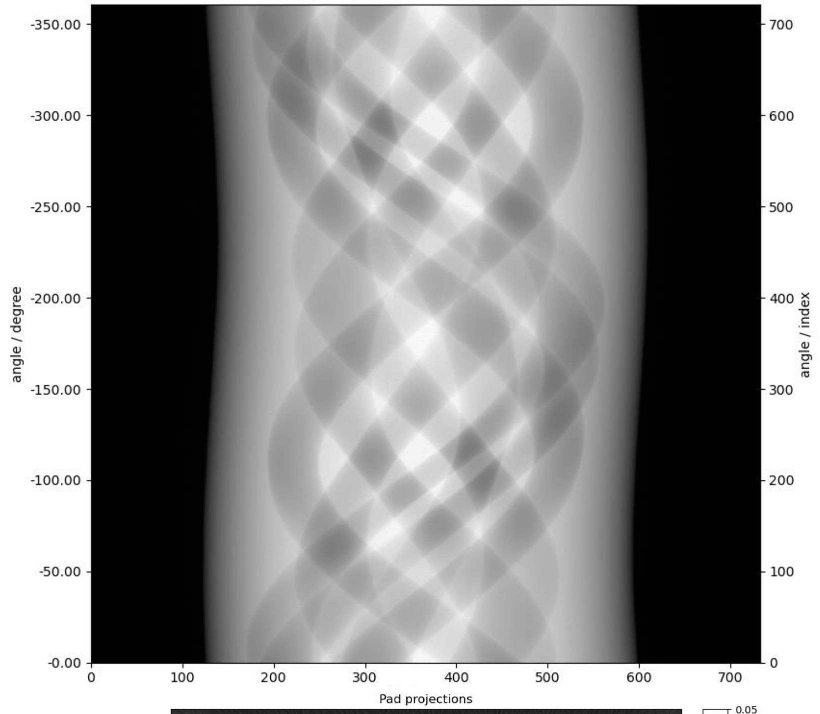
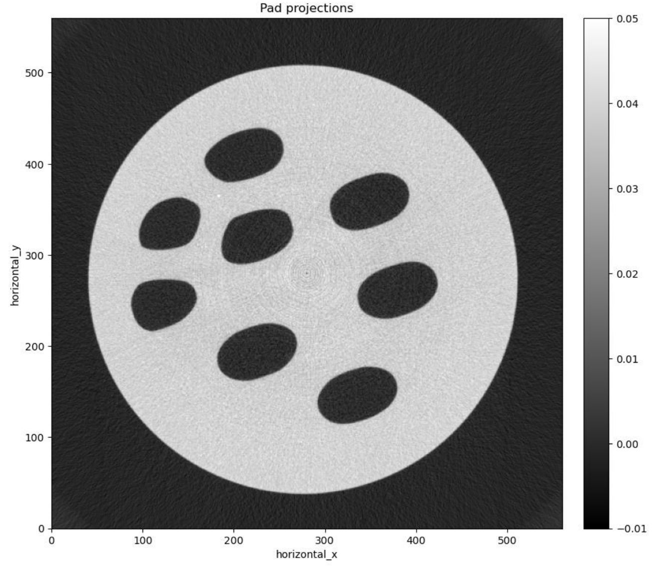






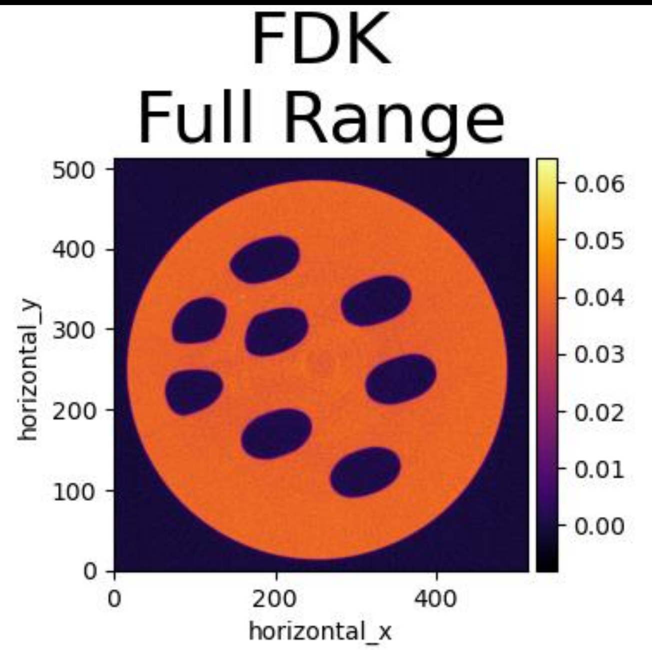
Reconstruction: Exploit prior knowledge

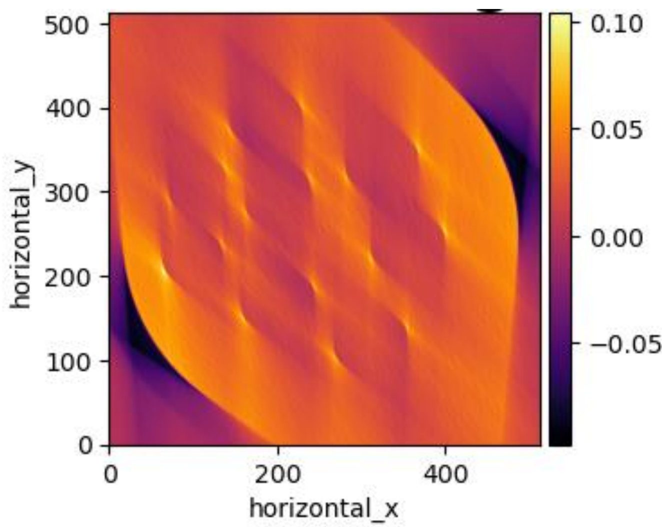
Construct optimization problem to express what we know:

- Single homogeneous material
- Sharp edges

- Object is approximately disk shaped
- Zero attenuation outside the object
- Constant value of 0.0409 mm^{-1} inside the object

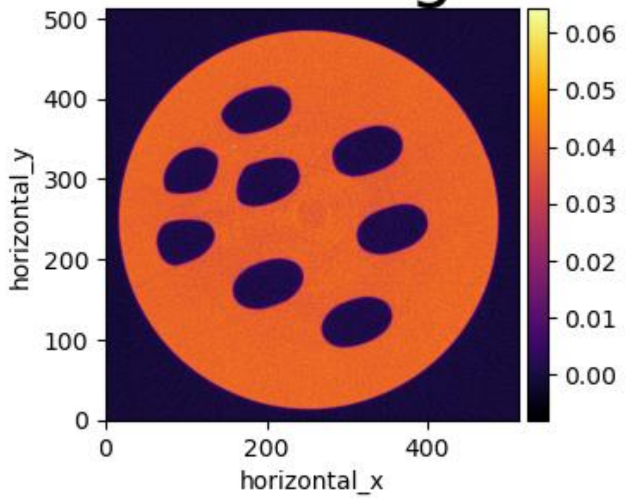
- Edges perpendicular to projection angles are the most difficult (micro-local analysis)





Prior knowledge: homogeneous material with sharp edges

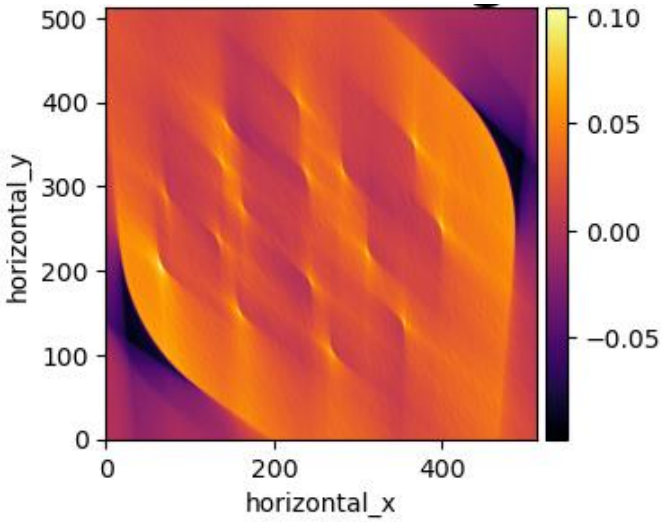
FDK
Full Range



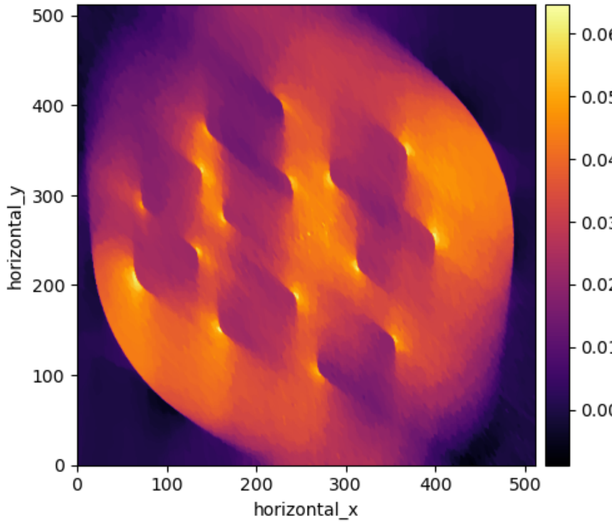
$$\min_{\mathbf{u}} \quad a_1 \|\mathbf{A}\mathbf{u} - \mathbf{b}\|_2^2 + a_2 \mathrm{ITV}(\mathbf{u})$$

LS + isoTV   Mask: none 50 deg.   Score: 0.713

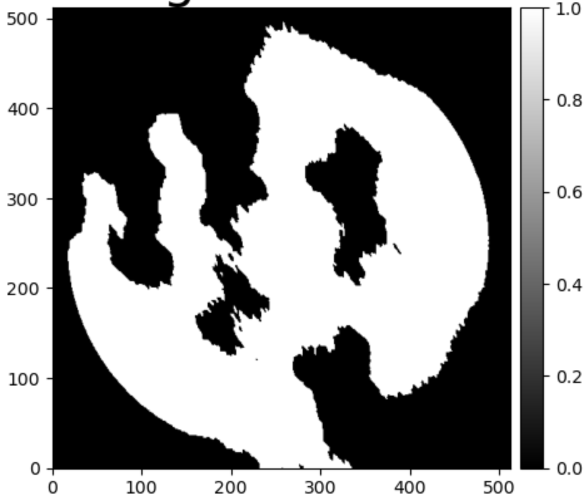
FDK Score: 0.430



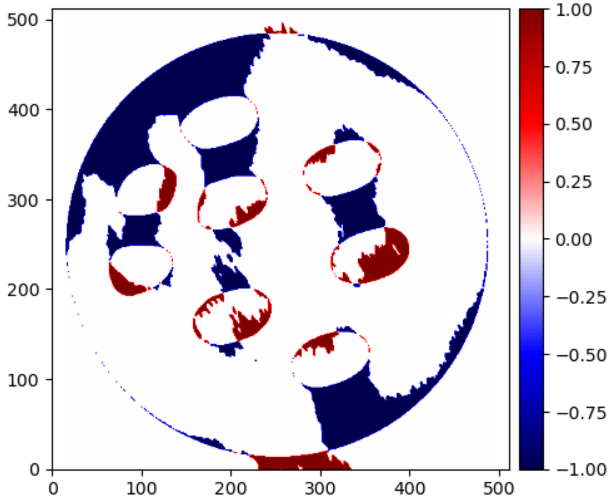
Reconstruction



Segmentation

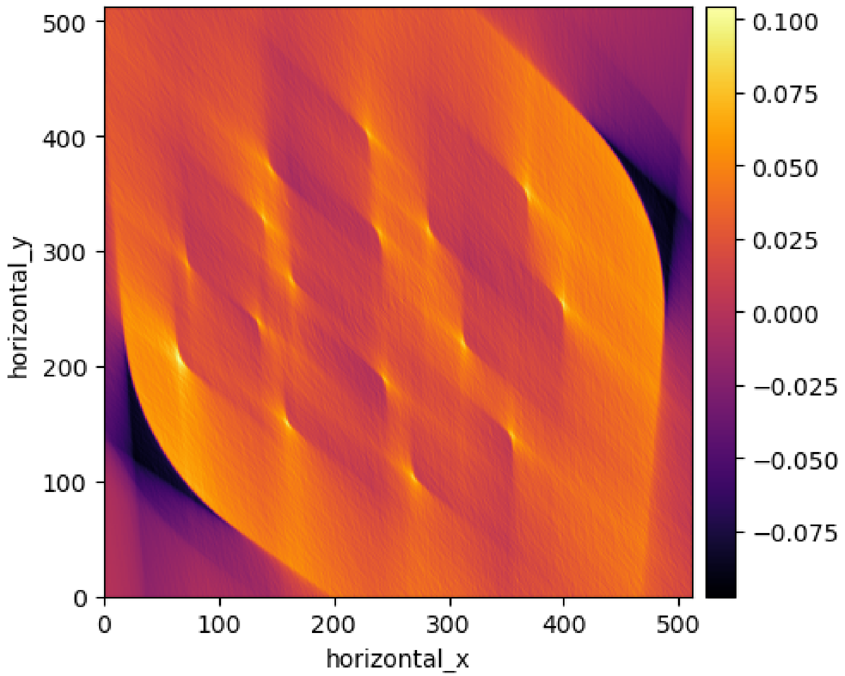


Diff

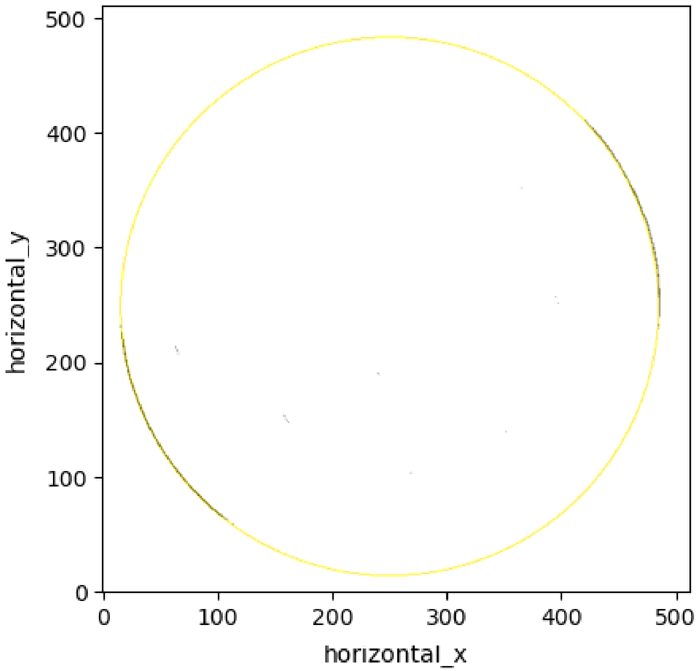


Prior knowledge: approximately disk shaped

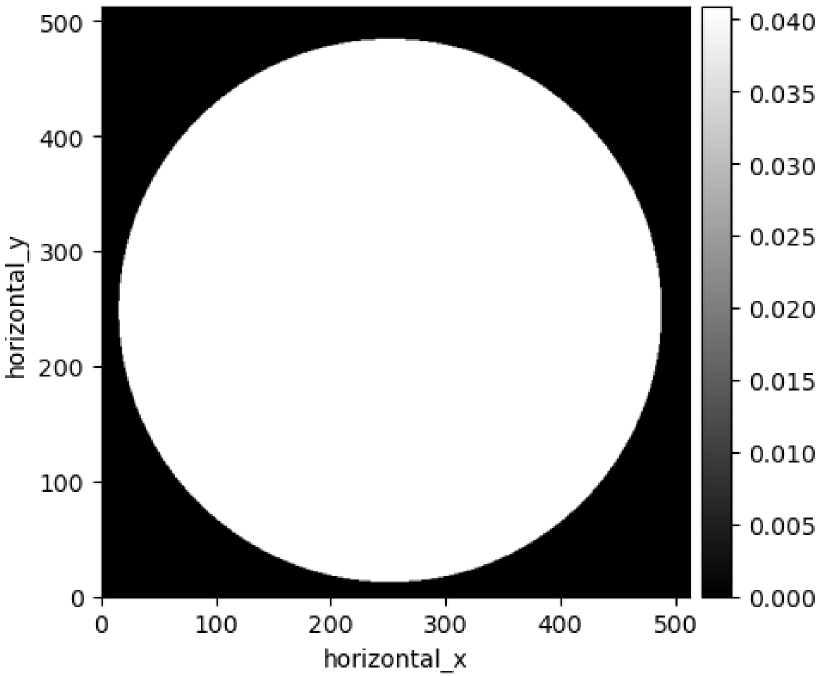
Limited angle FDK



Gradient Magnitude



Disk

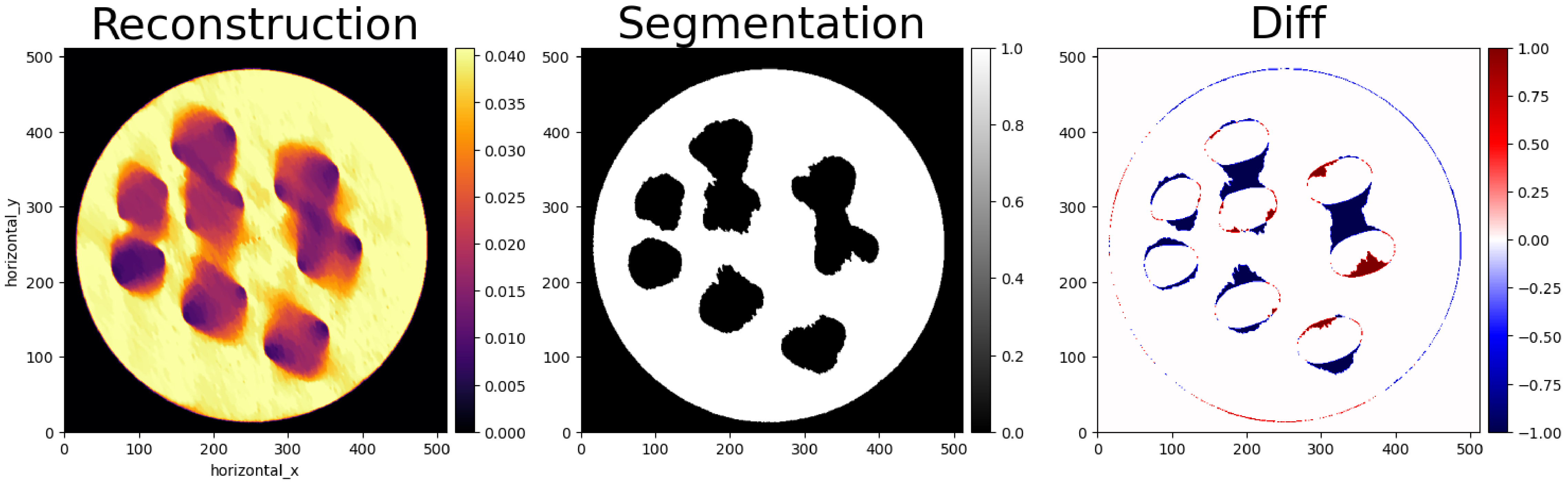


I.D. Coope *Circle fitting by linear and nonlinear least squares in 2D*
`https://link.springer.com/article/10.1007/BF00939613`

Disk shape with known attenuation as constraints

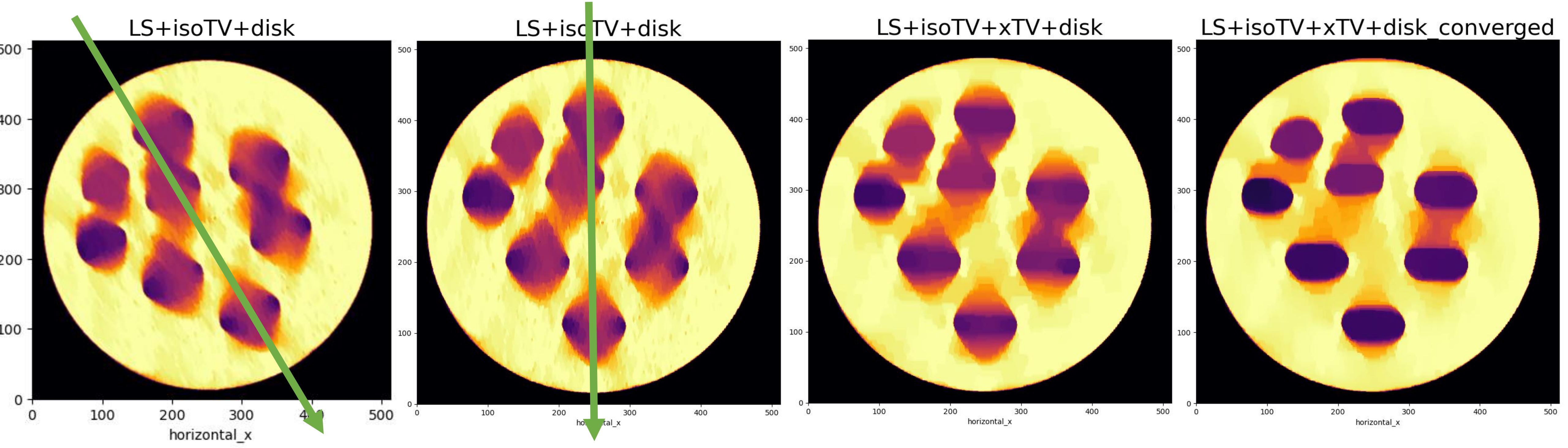
$$\min_{\mathbf{u}} \quad a_1\|\mathbf{A}\mathbf{u}-\mathbf{b}\|_2^2 + a_2 \mathrm{ITV}(\mathbf{u})$$
$$\text{s.t.} \quad \mathbf{0} \leq \mathbf{u} \leq v\mathbf{m}$$

LS + isoTV   Mask: fitted 50 deg.   Score: 0.919



Anisotropic TV

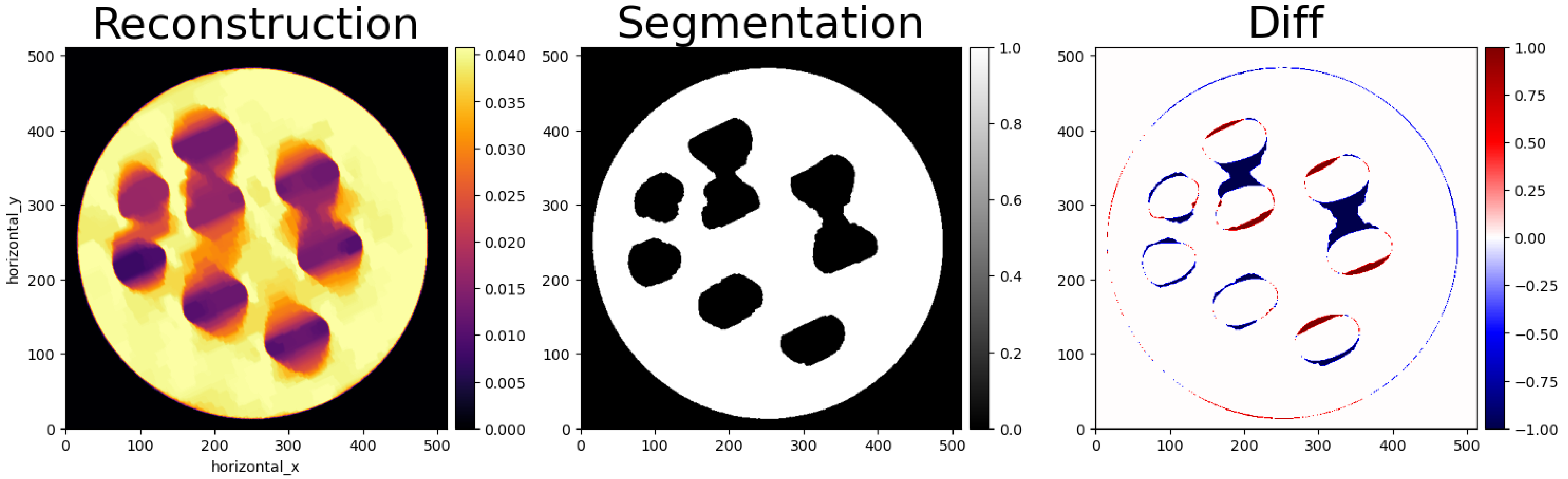
- Blurred edges along central projection direction
- Rotate to align with coordinate axis
- Apply single-directional TV to encourage edges in blurred direction
- Remember to check convergence!
- Rotate back



LS + isoTV + disk + xTV

$$\min_{\mathbf{u}} \quad a_1\|\mathbf{Au}-\mathbf{b}\|_2^2 + a_2\mathrm{ITV}(\mathbf{u}) + a_3\mathrm{ATV}_x(\mathbf{u})$$
$$\text{s.t.} \quad \mathbf{0} \le \mathbf{u} \le v\mathbf{m}$$

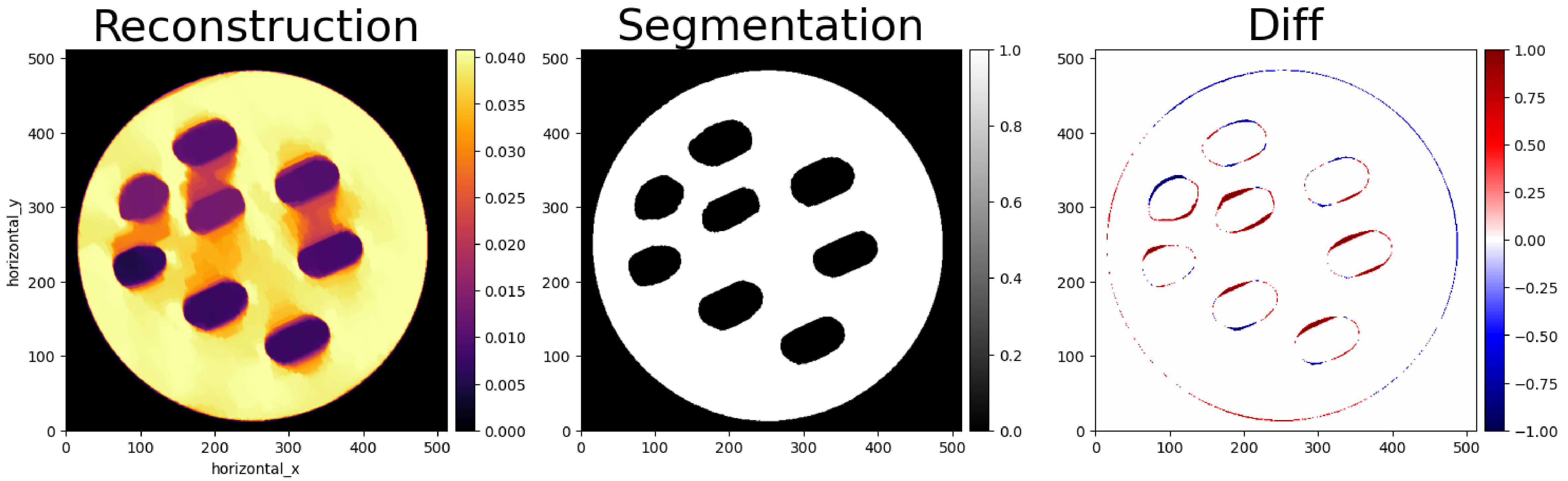
LS + isoTV + xTV   Mask: fitted 50 deg.   Score: 0.934



LS + isoTV + disk + xTV (converged)

$$\begin{aligned} \min_{\mathbf{u}} \quad & a_1\|\mathbf{A}\mathbf{u}-\mathbf{b}\|_2^2 + a_2\mathrm{ITV}(\mathbf{u}) + a_3\mathrm{ATV}_x(\mathbf{u}) \\ \text{s.t.} \quad & \mathbf{0} \leq \mathbf{u} \leq v\mathbf{m} \end{aligned}$$

Converged LS + isoTV + xTV Mask: fitted 50 deg. Score: 0.973



Primal Dual Hybrid Gradient (PDHG) method in CIL

CIL offers a range of optimization algorithms, incl GD, FISTA, ADMM and **PDHG:**

$$\min_{\mathbf{u}} \quad f(\mathbf{K}\mathbf{u}) + g(\mathbf{u})$$

$$\text{where} \quad f(\mathbf{K}\mathbf{u}) = \sum_i f_i(\mathbf{K}_i\mathbf{u})$$

Rewrite our optimization problem for PDHG:

$$\min_{\mathbf{u}} \quad a_1\|\mathbf{A}\mathbf{u}-\mathbf{b}\|_2^2 + a_2\|\mathbf{D}\mathbf{u}\|_{2,1} + a_3\|\mathbf{D}_{\mathbf{x}}\mathbf{u}\|_1 + \chi_{[\mathbf{0},v\mathbf{m}]}(\mathbf{u})$$

$$f = \begin{pmatrix} a_1\|\cdot - \mathbf{b}\|_2^2 \\ a_2\|\cdot\|_{2,1} \\ a_3\|\cdot\|_1 \end{pmatrix} \quad \mathbf{K} = \begin{pmatrix} \mathbf{A} \\ \mathbf{D} \\ \mathbf{D}_{\mathbf{x}} \end{pmatrix} \qquad g = \chi_{[\mathbf{0},v\mathbf{m}]}$$

Solving with CIL – "near-math" syntax

$$f = \begin{pmatrix} a_1\|\cdot - \mathbf{b}\|_2^2 \\ a_2\|\cdot\|_{2,1} \\ a_3\|\cdot\|_1 \end{pmatrix}$$

```
F = BlockFunction( a1*L2NormSquared(data),
                   a2*MixedL21Norm(),
                   a3*L1Norm() )
```

$$\mathbf{K} = \begin{pmatrix} \mathbf{A} \\ \mathbf{D} \\ \mathbf{D_x} \end{pmatrix}$$

```
K = BlockOperator( ProjectionOperator(ig, ag),
                   GradientOperator(ig),
                   FiniteDifferenceOperator(ig,'horizontal_x') )
```

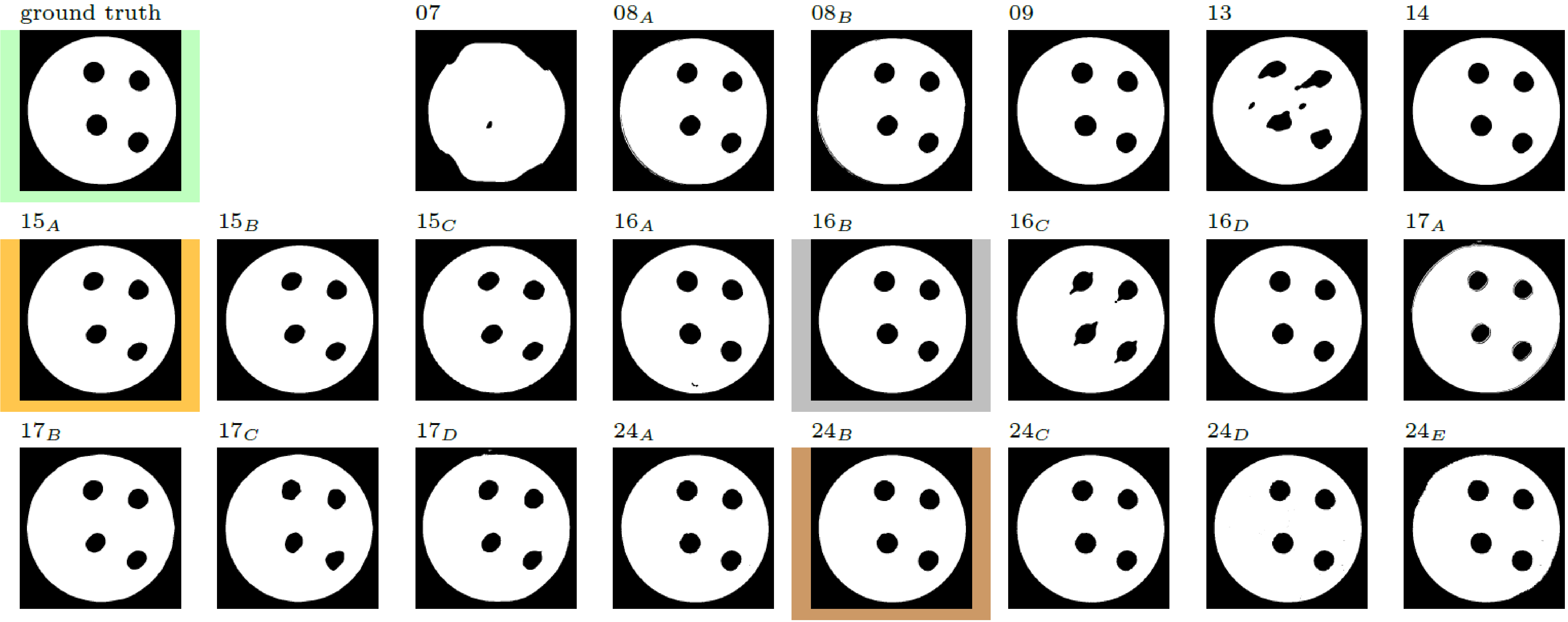
$$g = \chi_{[\mathbf{0},v\mathbf{m}]}$$

```
G = IndicatorBoxPixelwise( lower=0.0,
                           upper=v*m )
```

$$\min_{\mathbf{u}} \quad f(\mathbf{Ku}) + g(\mathbf{u})$$

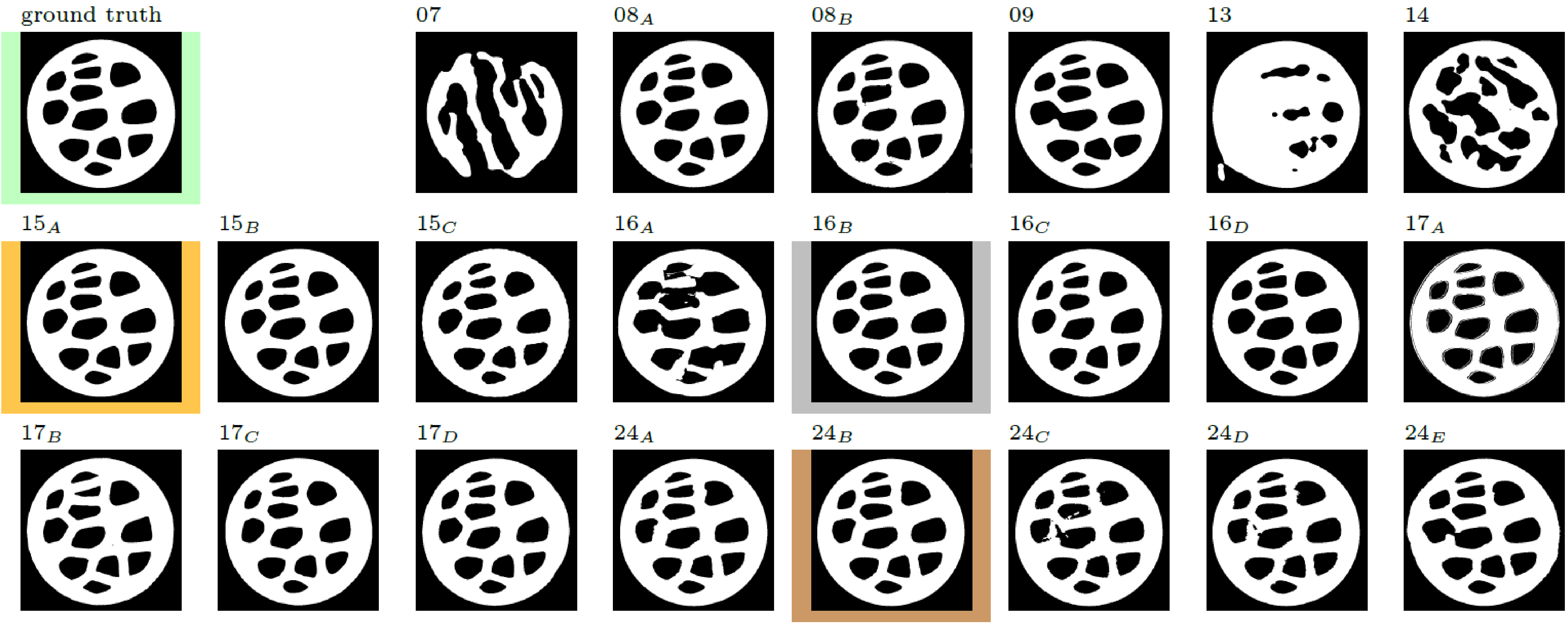
```
algo = PDHG( initial=ig.allocate(0.0),
             f=F,
             g=G,
             operator=K,
             max_iteration=2000 )
algo.run()
```

Level 1: 90 deg

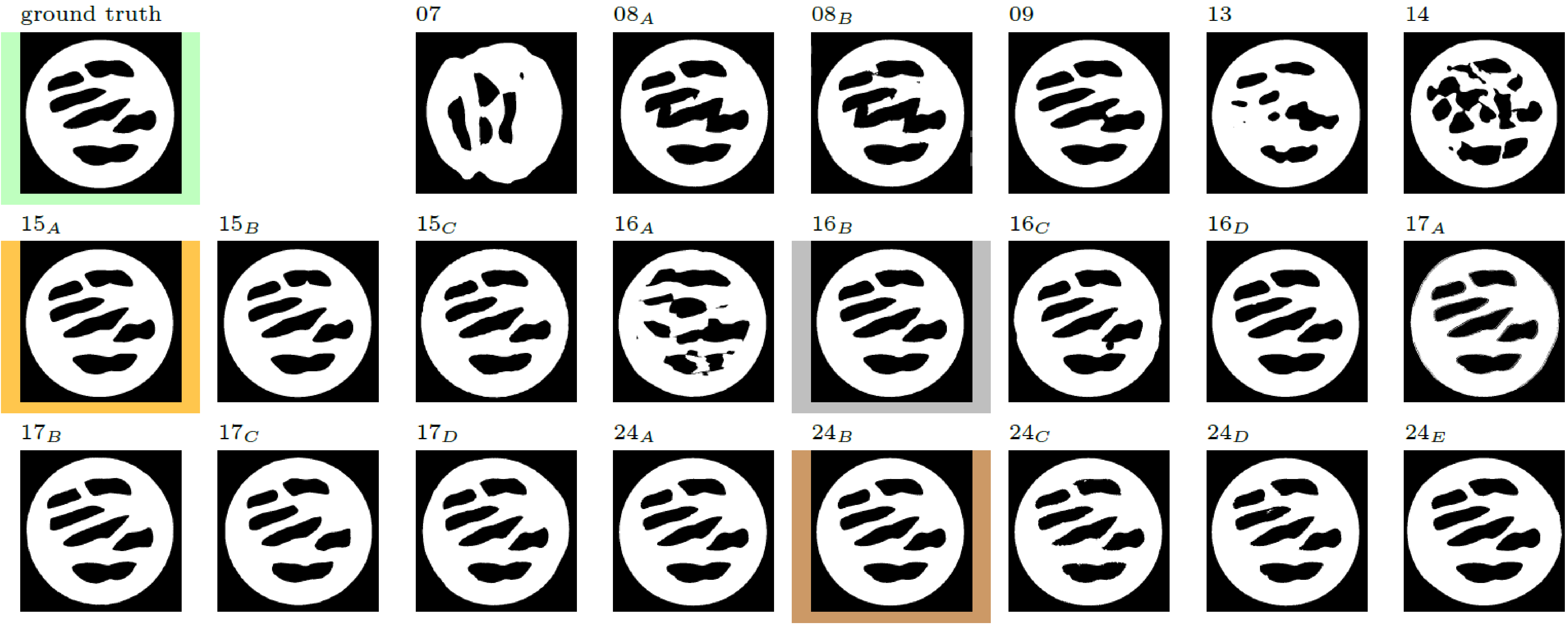


Level 2: 80 deg

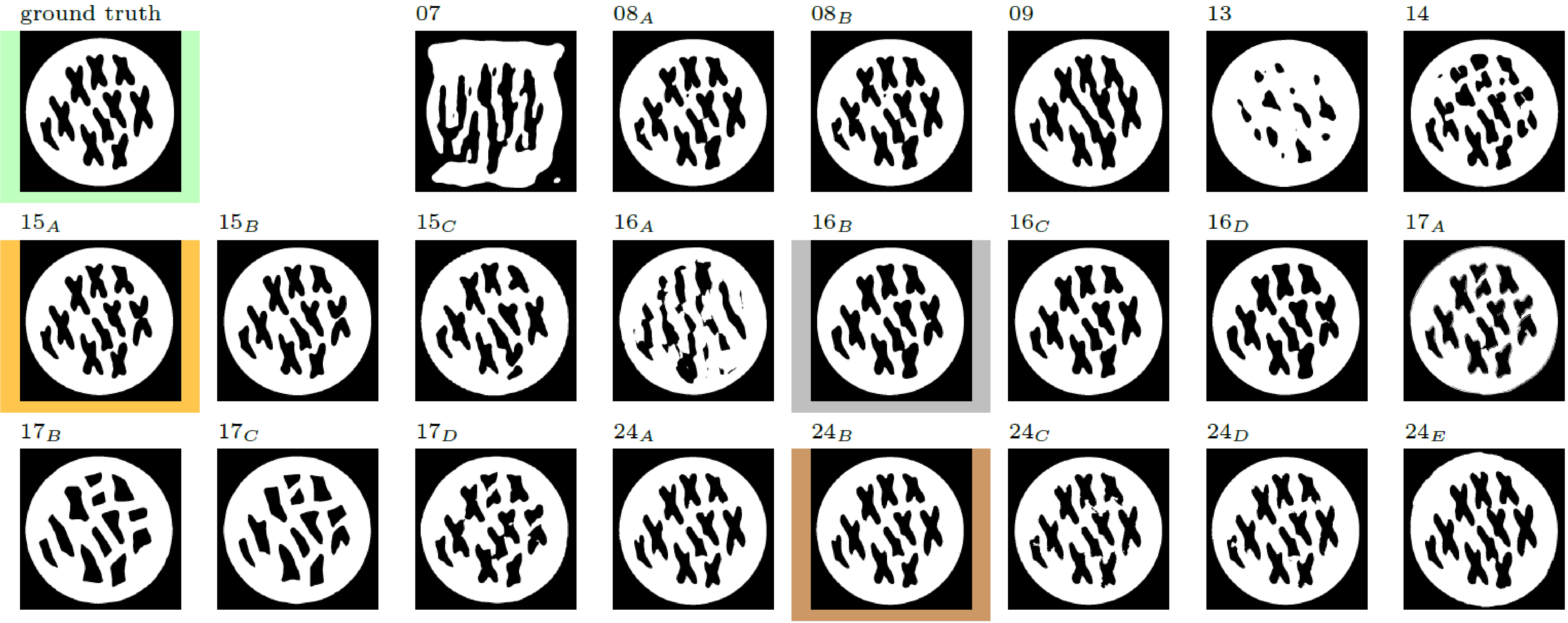
Level 3: 70 deg



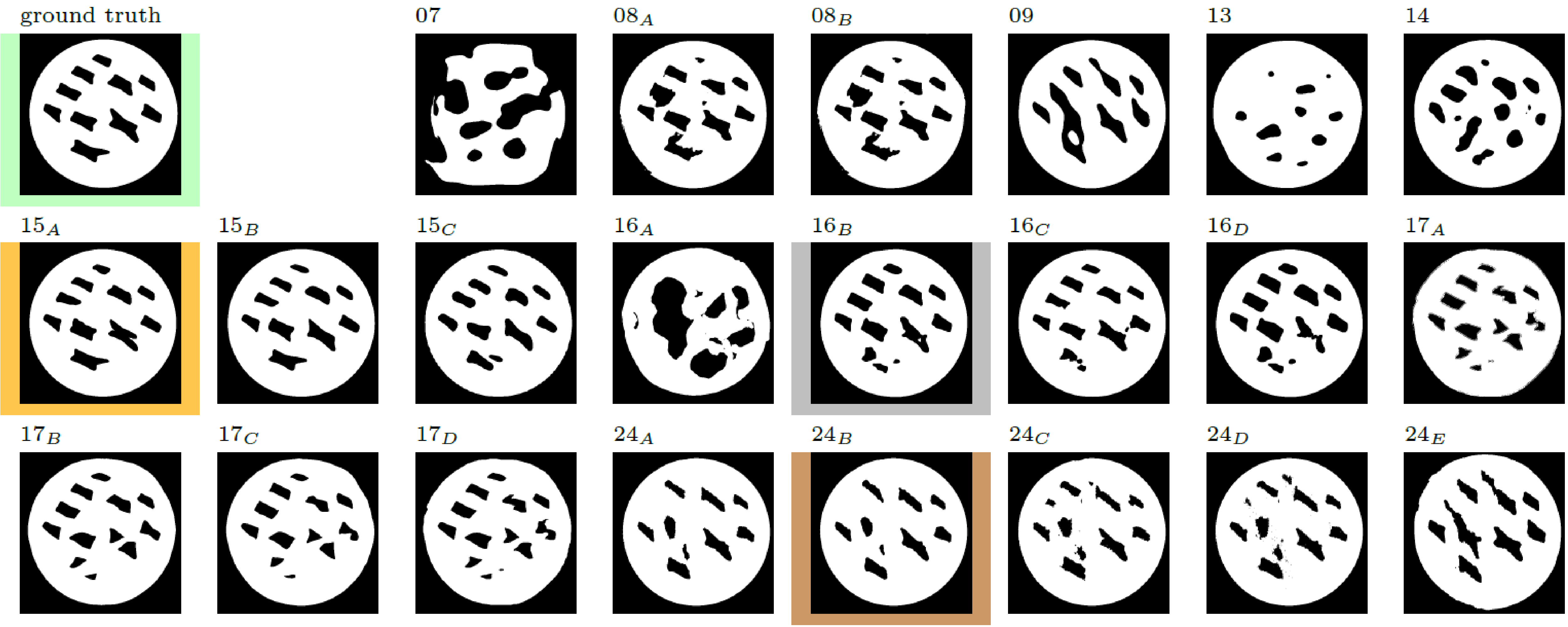
Level 4: 60 deg



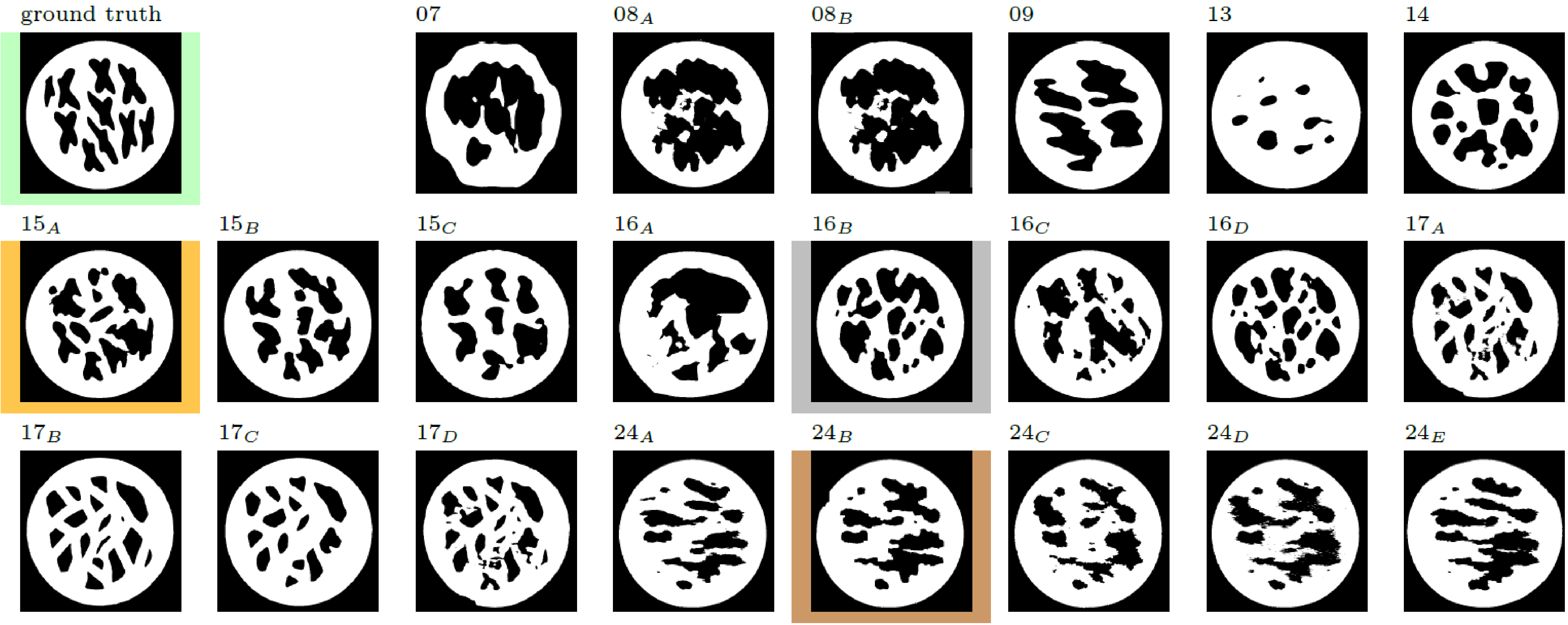
Level 5: 50 deg



Level 6: 40 deg



Level 7: 30 deg



Ideas to explore if more time

- Improved segmentation – area we spent least time
- Ensure converged solution!
- Enforce acrylic value in outermost circular band
- Combine with L1-norm sparsity regularizer to force zero values

Conclusions – thanks for your attention!

- Thanks to organizers – hope to see more challenges!

- CIL
  - HTC submission: github.com/TomographicImaging/CIL-HTC2022-Algo2
  - ArXiv preprint: https://arxiv.org/abs/2310.01671
  - Main site: ccpi.ac.uk/cil
  - Demos: github.com/TomographicImaging/CIL-Demos
  - Discord community: discord.gg/9NTWu9MEGq

Open in Colab

- Ongoing work and future plans
  - Deploy at facilities: ESRF, Diamond, NXRF, DTU 3DIM, ISIS, …
  - More reconstruction methods – talk to us!
  - New modalities – talk to us!
  - User training and hackathon events
  - jakj@dtu.dk

CCPi TOMOGRAPHIC IMAGING

CIL CORE IMAGING LIBRARY