
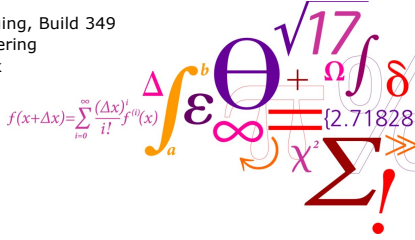


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Reconstruction in CT and relation to other imaging modalities


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 October 26, 2023

Center for Fast Ultrasound Imaging, Build 349
 Department of Electrical Engineering
 Technical University of Denmark

$$f(x+\Delta x) = \sum_{i=0}^{\infty} \frac{(\Delta x)^i}{i!} f^{(i)}(x)$$


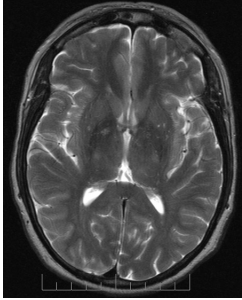
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Reconstruction - outline

- Fan-beam geometry and reconstruction
- Overview of other reconstruction methods
 - In the Fourier domain - MR scanning
 - Algebraic reconstruction
 - PET and PET/CT scanning
- Reading material: Prince & Links chapter 6 & 9

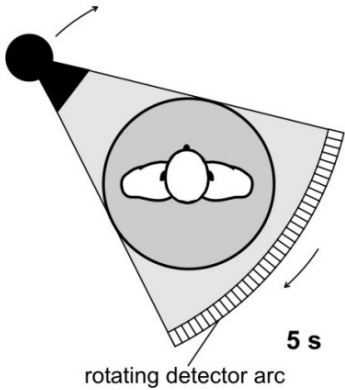


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Fan beam scan




rotating detector arc
5 s

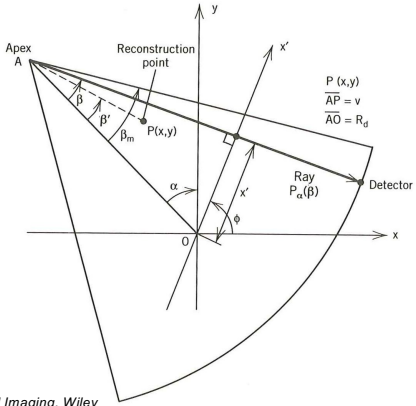
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From: W. A. Kalender: Computed Tomography, Publicis, 2005

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Fan beam reconstruction geometry



From Cho et al (1993),
 Foundations of Medical Imaging, Wiley

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Fan-beam reconstruction algorithm



$$\hat{f}(x, y) = \int_0^{2\pi} w_2 \left[\int_{-\beta_m}^{\beta_m} w_1 p_\alpha(\beta) g(\beta' - \beta) d\beta \right] d\alpha$$

- Weight 1: $w_1 = R_d \cos \beta$
- Weight 2: $w_2 = \frac{1}{2\pi} \frac{1}{v^2}$
- Filter: $g(\beta) = \frac{\beta}{\sin \beta} h(\beta)$, $h(\beta) \leftrightarrow |\rho|$
- Definition of variables on previous slide

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Reconstruction methods



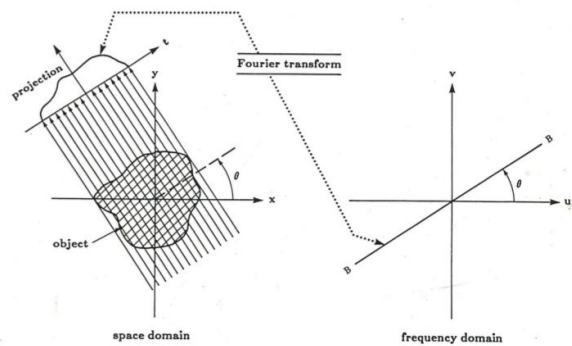
Table 3-1 Image reconstruction algorithms

Projection Reconstruction (PR)	2-D PR	Filtered Backprojection (FB)	Parallel-Beam Mode Fan-Beam Mode
		Backprojection Filtering (BF)	Parallel-Beam or Fan-Beam Mode
	3-D PR	True Three-Dimensional Reconstruction (TTR)	Parallel-Beam Mode Cone-Beam Mode
		Generalized TTR (GTTR)	
		Planar-Integral Projection Reconstruction (PPR)	
Iterative Method	Algebraic Reconstruction Technique (ART)		
	Maximum Likelihood Reconstruction (MLR) or Expectation Maximization (EM) Reconstruction		
Fourier Reconstruction (FR)	Direct Fourier Reconstruction (DFR)		
	Direct Fourier Imaging in NMR		

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From Cho et al (1993).
Foundations of Medical Imaging, Wiley

Fourier slice theorem



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Reconstruction in the Fourier domain

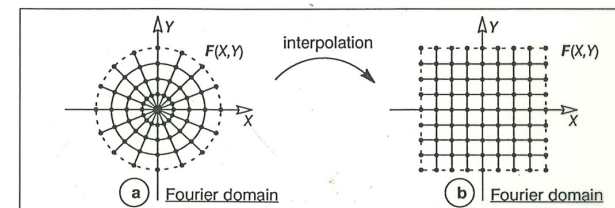


Fig. 2.15 a) The Fourier transform of the projections gives values along radial lines in the Fourier domain of the object. (a-b) Interpolation is used to get values on a square grid.

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From Magnusson (1993).
Linogram and other direct Fourier methods
for tomographic reconstruction. Linköping

MR scanner



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Magnetic Resonance (MR) scanning

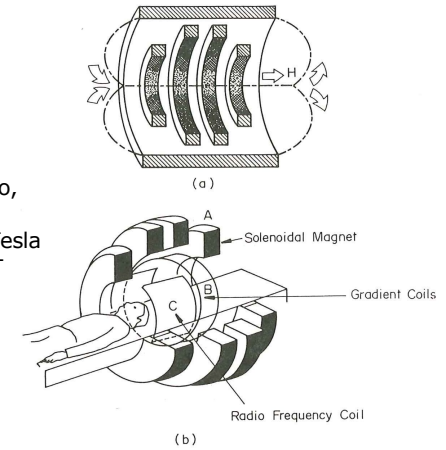


Larmor frequency:

$$\omega_0 = \gamma B_0$$

γ – Gyromagnetic ratio,
42.58 MHz/Tesla

B_0 – Magnetic field in Tesla
Typically 1.5 – 3 T



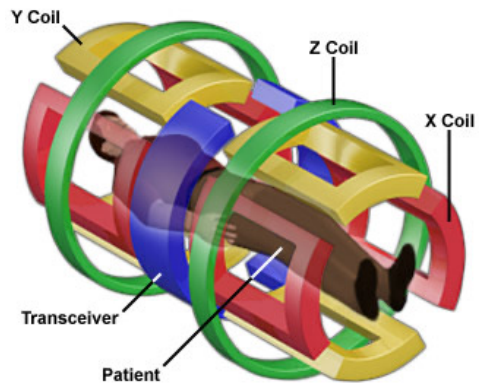
From Cho et al (1993),
Foundations of Medical Imaging, Wiley

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MRI Scanner Gradient Magnets

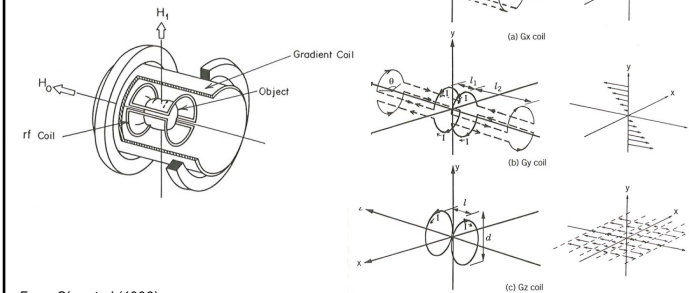


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Gradient coils



From Cho et al (1993),
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MR measurement and reconstruction

the k-space, $F(k_x, k_y)$

$s(t)$ measures one row of data in the k-space

Fig. 1.17 The measured signal $s(t)$ measures one row in the 2D Fourier domain $F(k_x, k_y)$ of the nuclei density $f(x, y)$.

z-gradient ΔB_z

y-gradient ΔB_y

x-gradient ΔB_x

FID-measure and readout of different k_y -rows:

Fig. 1.18 A time diagram for the application of magnetic field gradients. Readout of several rows k_y in the Fourier domain of nuclei density.

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MR images

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MR overview image

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Algebraic reconstruction

$S_1 = \mu_1 + \mu_2$
 $S_2 = \mu_3 + \mu_4$
 $S_3 = \mu_1 + \mu_3$
 $S_4 = \mu_2 + \mu_4$

$S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}$

From: W. A. Kalender: Computed Tomography, Publicis, 2005

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PET and PET/CT scanning Positron Emission Tomography

Annihilation

Image Reconstruction

- Radioactive FDG-18 injected
- Radioactive decay gives positron
- Annihilation with electron yields two 511 keV photons (gamma rays)
- Detected along line of response

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Positron Emission Tomography

Site of radioactive decay

Positron path

Annihilation

Electron path

511 keV photon

511 keV photon

Detector

Detector

Coincidence detection circuit

Event: Yes or no?

From Prince & Links, 2015

Figure 9.4
Coincidence detection due to positron decay and annihilation.

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Images

CT

PET

PET/CT

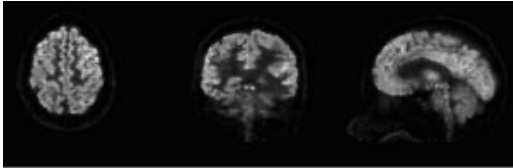
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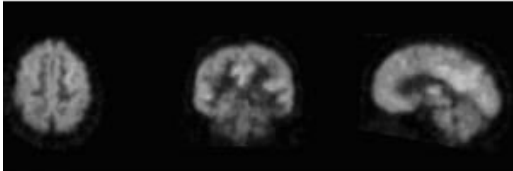
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HRRT PET scanner with ART

HRRT scanner



Conventional PET scanner



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Reconstruction methods

Table 3-1 Image reconstruction algorithms

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			Fan-Beam Mode
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From Cho et al (1993).
Foundations of Medical Imaging, Wiley

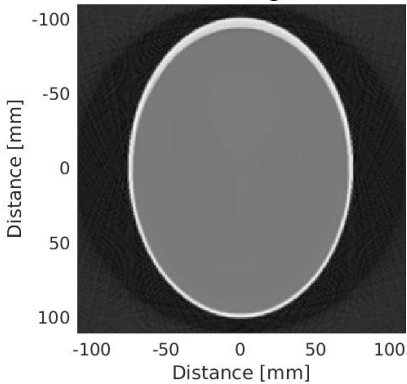
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Exercise 5 - Shepp-Logan phantom

Full range



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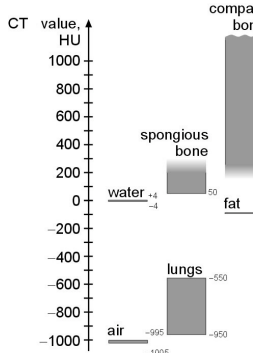
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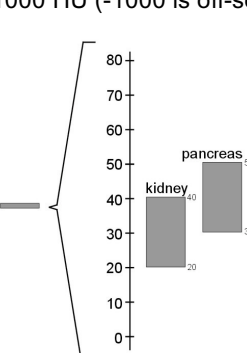
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Hounsfield units

Note that the *in-vivo* projected data on the website is off-set by 1000 HU (-1000 is off-set to 0)



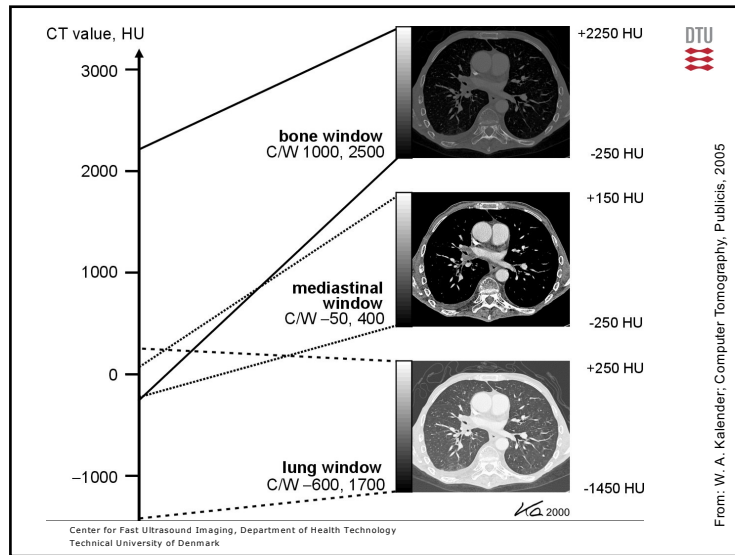


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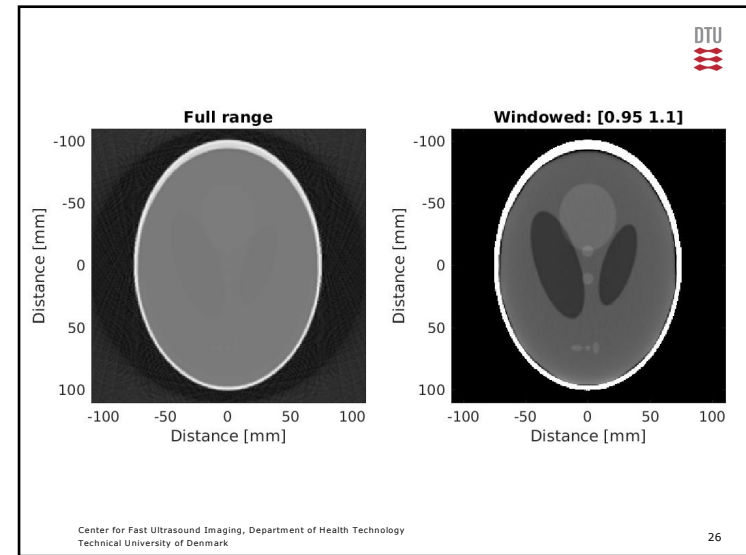
From: W. A. Kalender: Computed Tomography, Publicis, 2005
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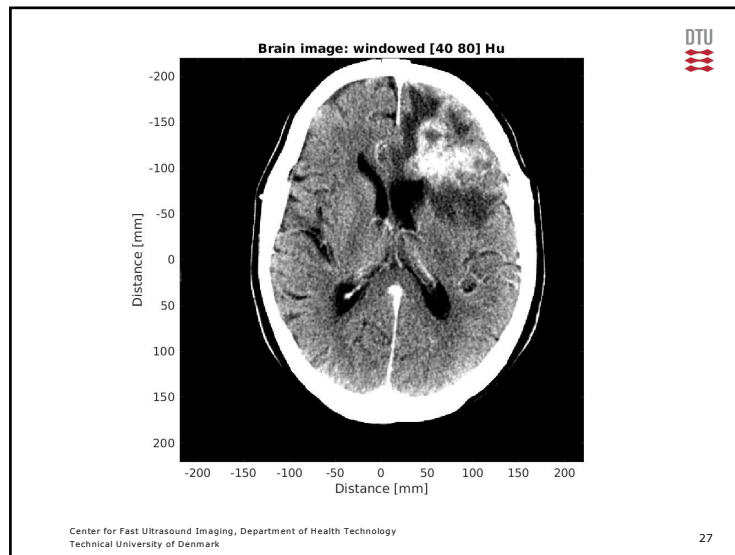
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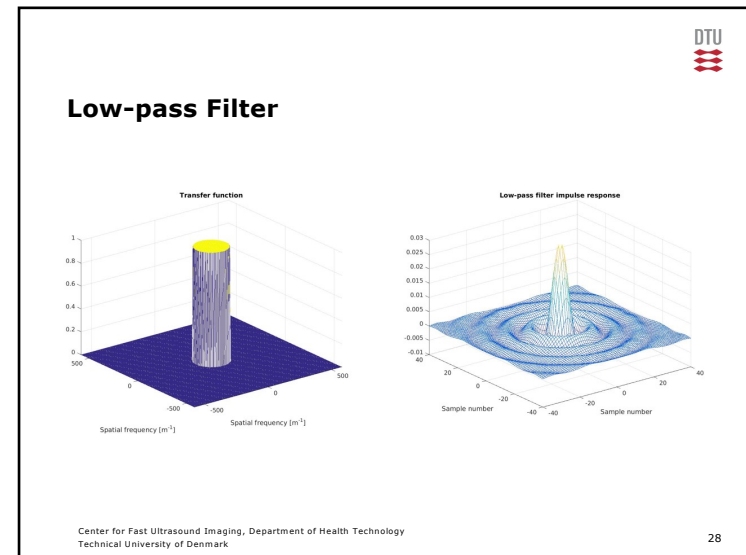
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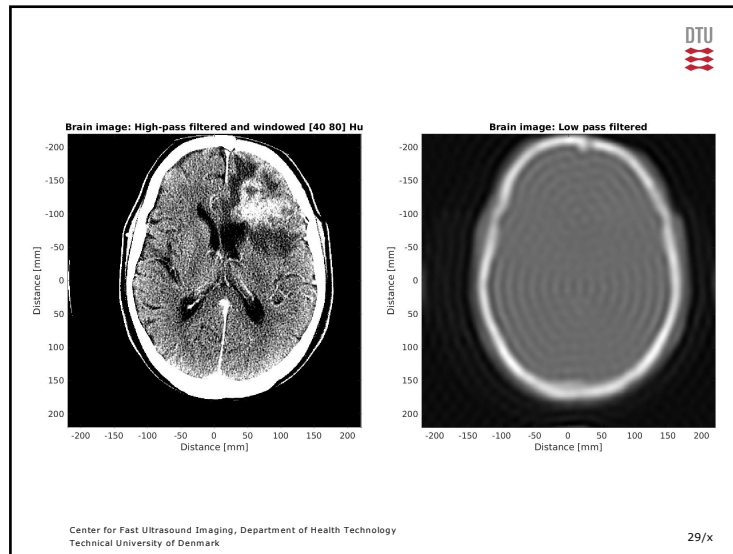
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Reconstruction summary

- Filtered backprojection algorithm and choices
- Fan-beam geometry and reconstruction
- Overview of other reconstruction methods –
 - MR, PET, PET/CT
- Advise for the assignments

- Next time: PET imaging by Chief physicist Søren Holm, Clinical Physiology and Nuclear Medicine, Rigshospitalet
- Later Algebraic reconstruction with Senior researcher, PhD Jakob Sauer Jørgensen, DTU Compute
- Reading material:
 - Prince & Links chapter 6-9, 12 & 13



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