Reconstruction in CT and relation to other imaging modalities

Jørgen Arendt Jensen

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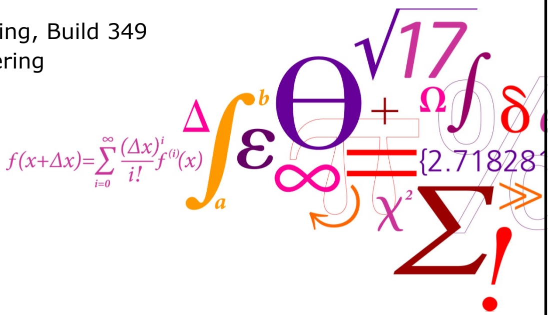
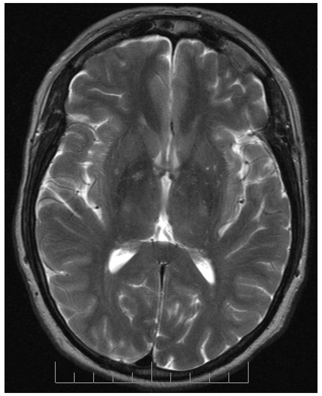
Center for Fast Ultrasound Imaging, Build 349
Department of Electrical Engineering
Technical University of Denmark



1

Reconstruction - outline

- Fan-beam geometry and reconstruction
- Overview of other reconstruction methods
  - In the Fourier domain – MR scanning
  - Algebraic reconstruction
    - PET and PET/CT scanning
- Reading material: Prince & Links chapter 6 & 9



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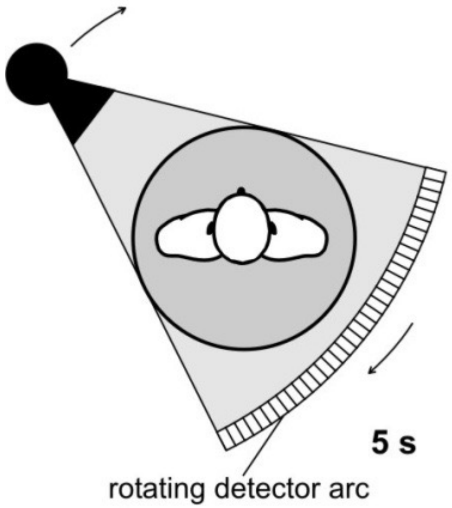
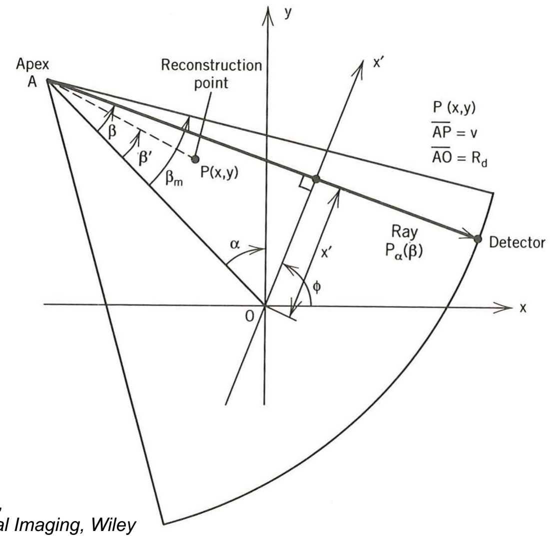
Fan beam scan



From: W. A. Kalender; Computed Tomography, Publicis, 2005

Fan beam reconstruction geometry



From Cho et al (1993),
Foundations of Medical Imaging, Wiley

Fan-beam reconstruction algorithm

$$\hat{f}(x,y) = \int_0^{2\pi} w_2 \left[\int_{-\beta_m}^{\beta_m} w_1 p_\alpha(\beta) g(\beta' - \beta) d\beta\right] d\alpha$$

- Weight 1: $w_1 = R_d \cos\beta$
- Weight 2: $w_2 = \frac{1}{2\pi}\frac{1}{v^2}$
- Filter: $g(\beta) = \frac{\beta}{\sin\beta} h(\beta), \quad h(\beta) \leftrightarrow |\rho|$
- Definition of variables on previous slide

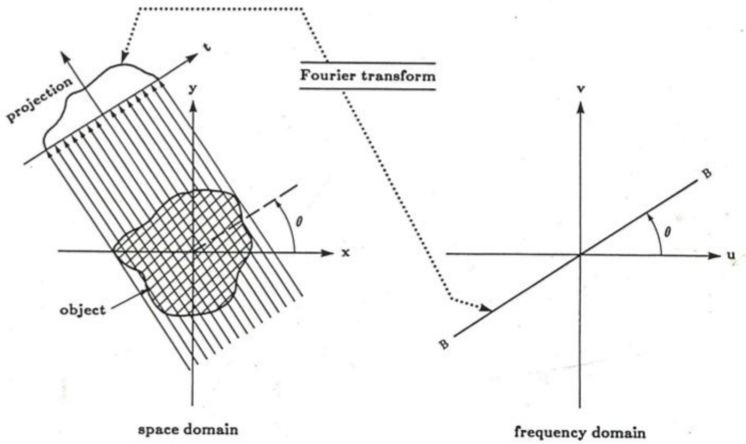
Reconstruction methods

Table 3-1 Image reconstruction algorithms

Projection Reconstruction (PR)	2-D PR	Filtered Backprojection (FB)	Parallel-Beam Mode
			Fan-Beam Mode
		Backprojection Filtering (BF)	Parallel-Beam or Fan-Beam Mode
	3-D PR	True Three-Dimensional Reconstruction (TTR)	Parallel-Beam Mode
			Cone-Beam Mode
		Generalized TTR (GTTR)	
		Planar-Integral Projection Reconstruction (PPR)	
Iterative Method	Algebraic Reconstruction Technique (ART)		
	Maximum Likelihood Reconstruction (MLR) or Expectation Maximization (EM) Reconstruction		
Fourier Reconstruction (FR)	Direct Fourier Reconstruction (DFR)		
	Direct Fourier Imaging in NMR		

From Cho et al (1993), Foundations of Medical Imaging, Wiley

Fourier slice theorem



Reconstruction in the Fourier domain

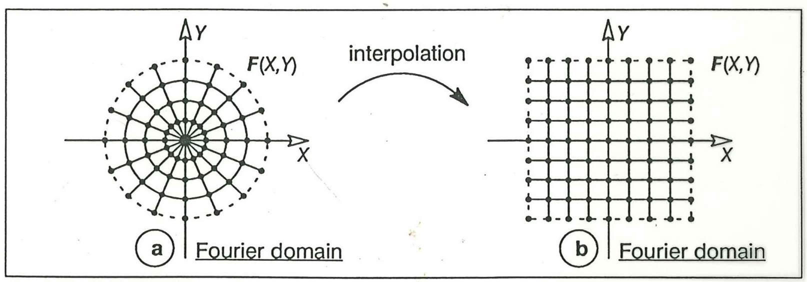


Fig. 2.15 **a)** The Fourier transform of the projections gives values along radial lines in the Fourier domain of the object. **(a–b)** Interpolation is used to get values on a square grid.

From Magnusson (1993), Linogram and other direct Fourier methods for tomographic reconstruction, Linköping

MR scanner

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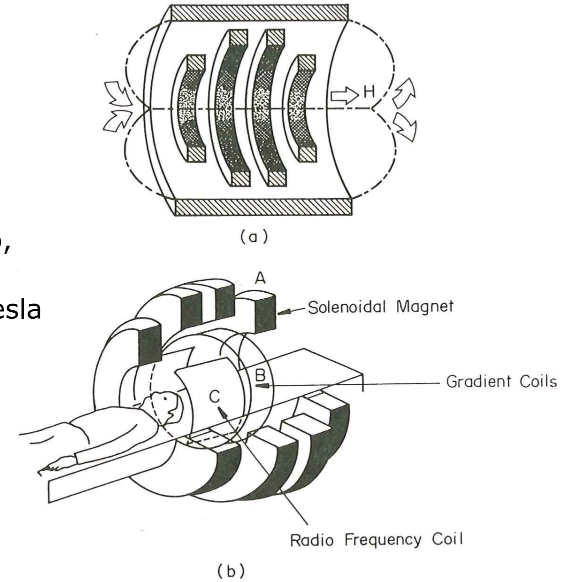
Magnetic Resonance (MR) scanning

Larmor frequency:

$$\omega_0 = \gamma B_0$$

γ – Gyromagnetic ratio, 42.58 MHz/Tesla

B_0 – Magnetic field in Tesla
Typically 1.5 – 3 T



From Cho et al (1993),
Foundations of Medical Imaging, Wiley

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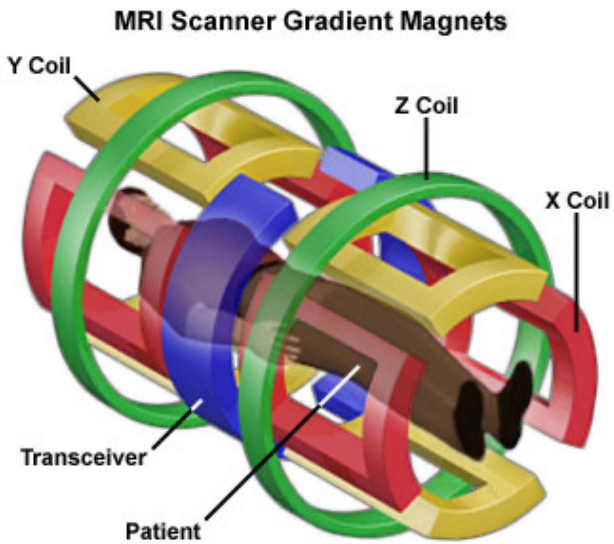

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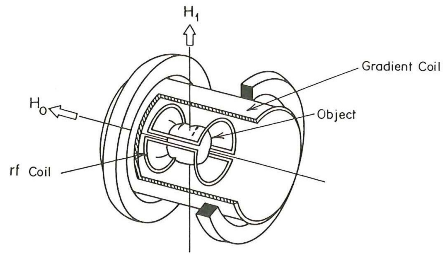
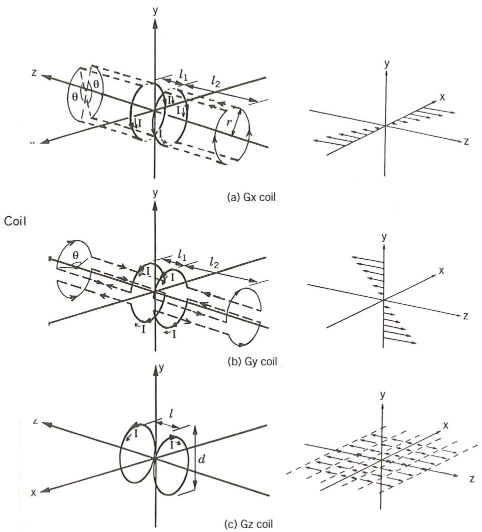
Gradient coils





From Cho et al (1993),
Foundations of Medical Imaging, Wiley

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MR measurement and reconstruction

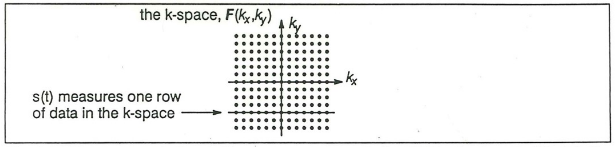


Fig. 1.17 The measured signal s(t) measures one row in the 2D Fourier domain $F(k_x,k_y)$ of the nuclei density $f(x,y)$.

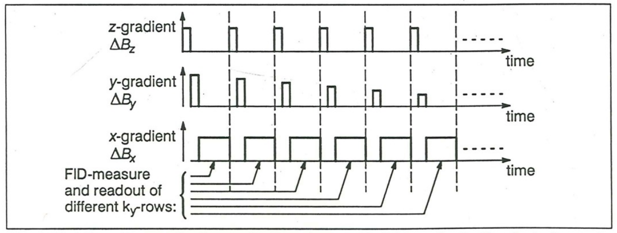
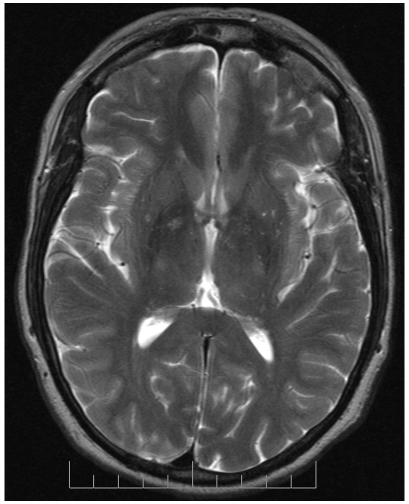
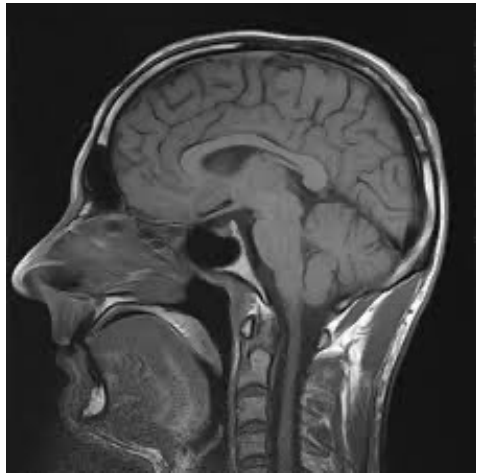


Fig. 1.18 A time diagram for the application of magnetic field gradients. Readout of *several* rows k_y in the Fourier domain of nuclei density.

MR images





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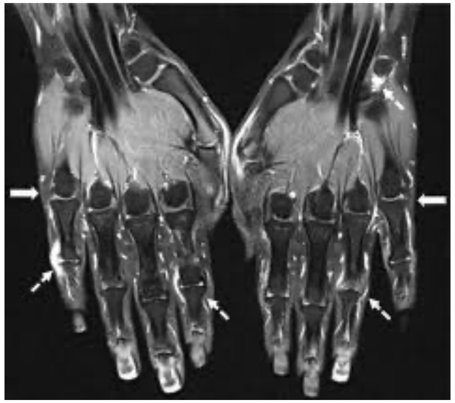
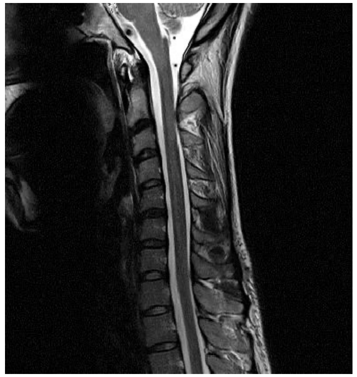
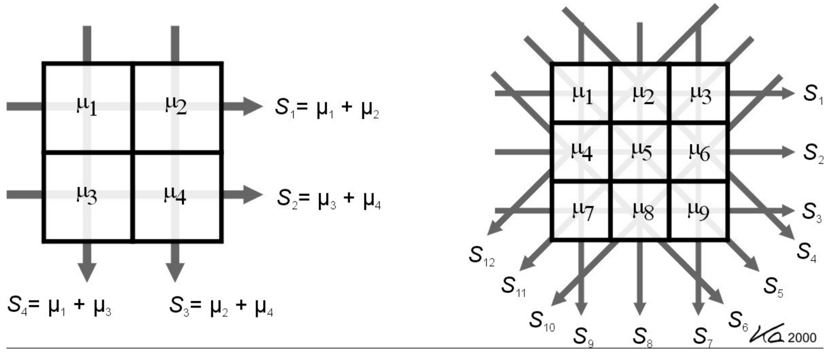


MR overview image



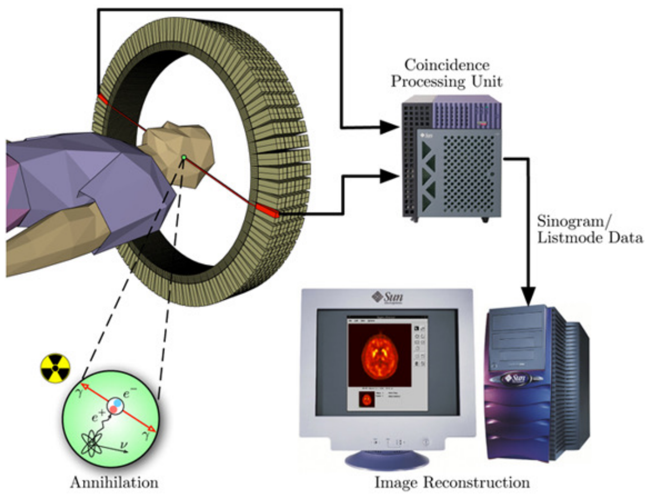
Algebraic reconstruction



From: W. A. Kalender; Computed Tomography, Publicis, 2005

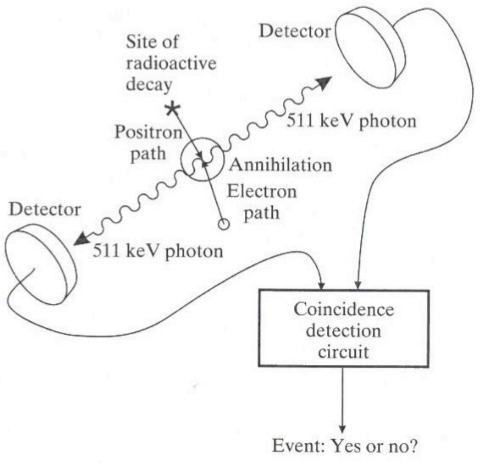
PET and PET/CT scanning

Positron Emission Tomography



- Radioactive FDG-18 injected
- Radioactive decay gives positron
- Annihilation with electron yields two 511 keV photons (gamma rays)
- Detected along line of response

Positron Emission Tomography

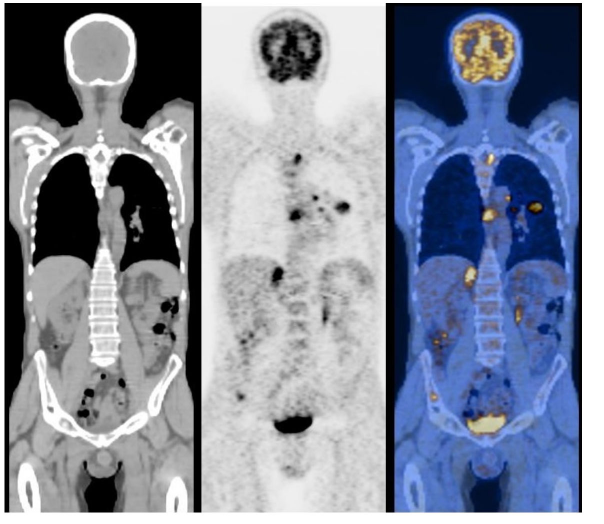


From Prince & Links, 2015

Figure 9.4
Coincidence detection due to positron decay and annihilation.

Images

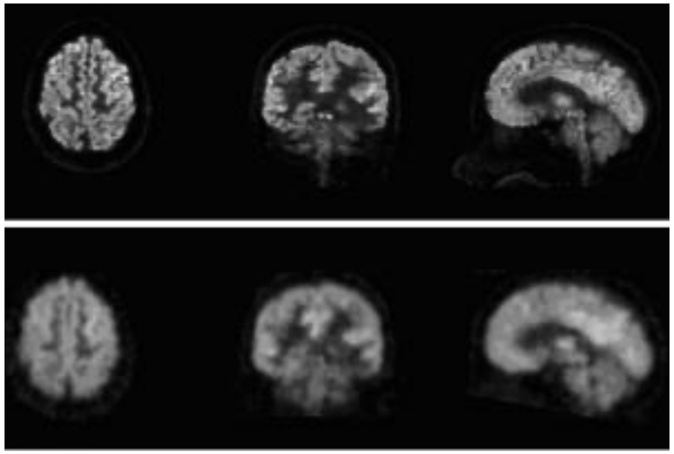
CT **PET** **PET/CT**



HRRT PET scanner with ART

HRRT scanner

Conventional PET scanner



Reconstruction methods

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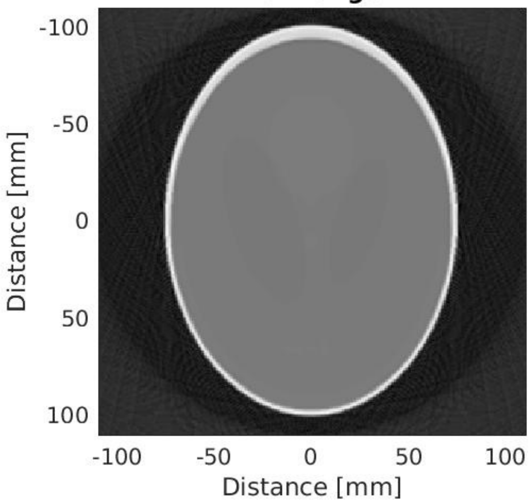
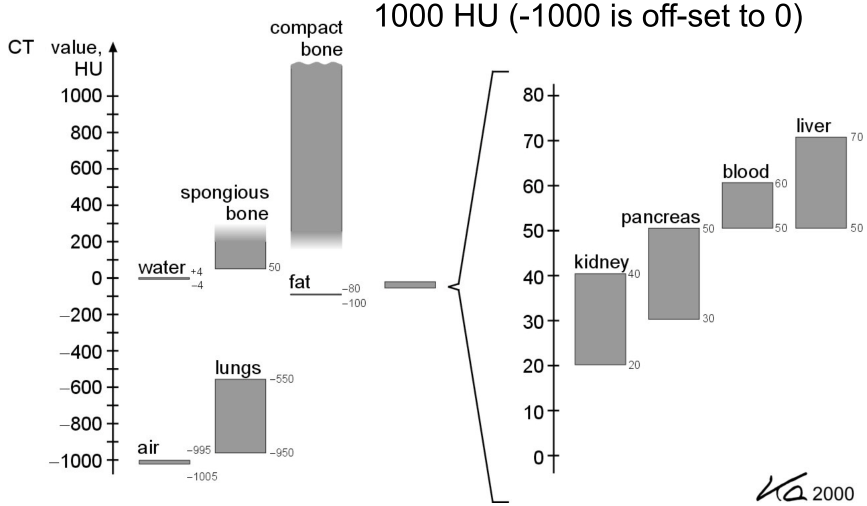
Exercise 5 - Shepp-Logan phantom



Hounsfield units

Note that the *in-vivo* projected data on the website is off-set by 1000 HU (-1000 is off-set to 0)



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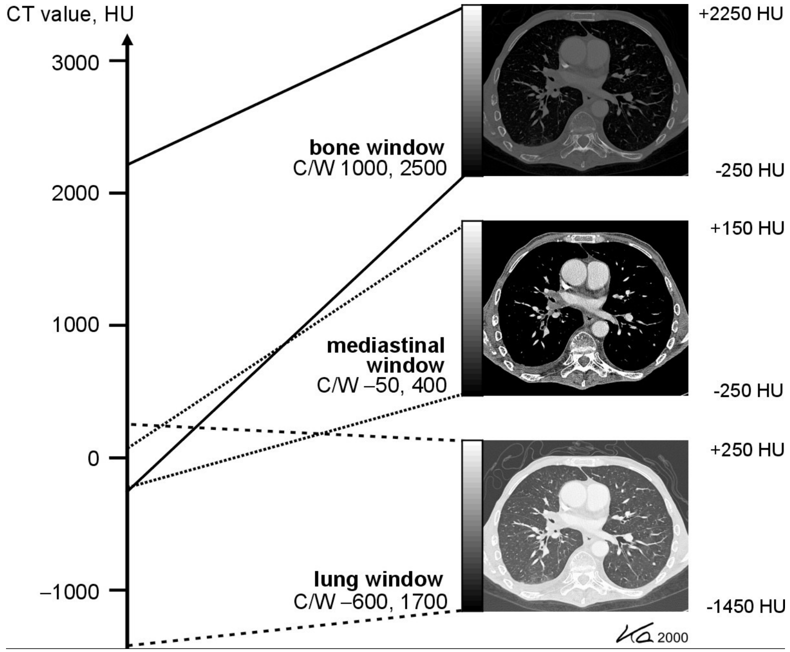

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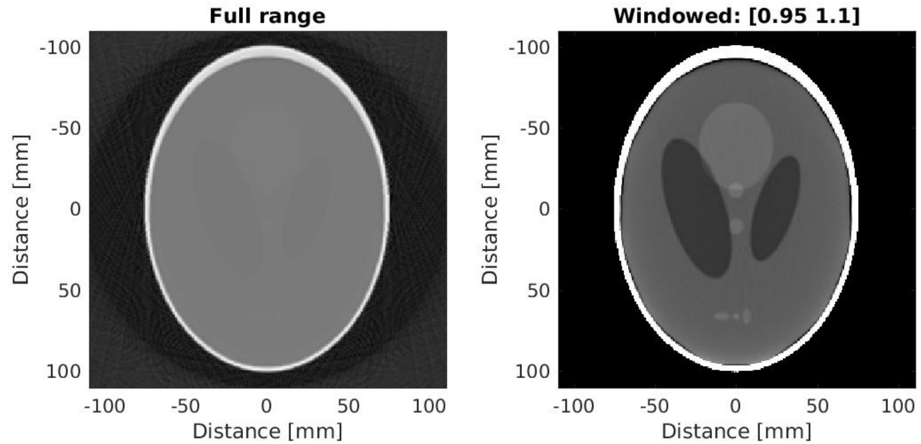
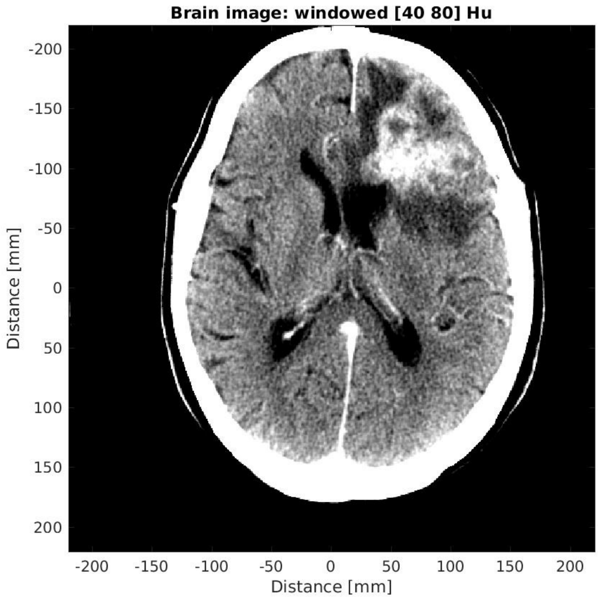
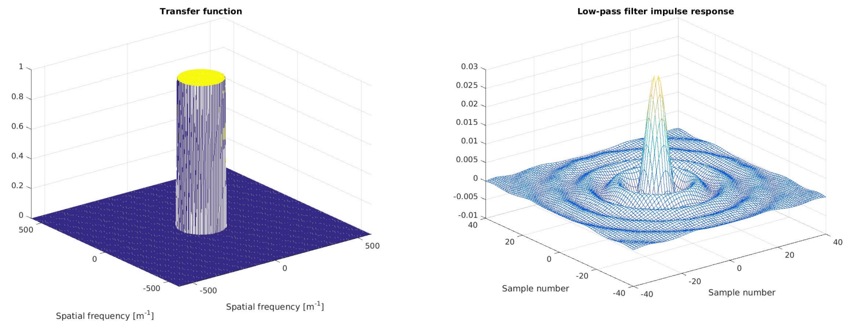
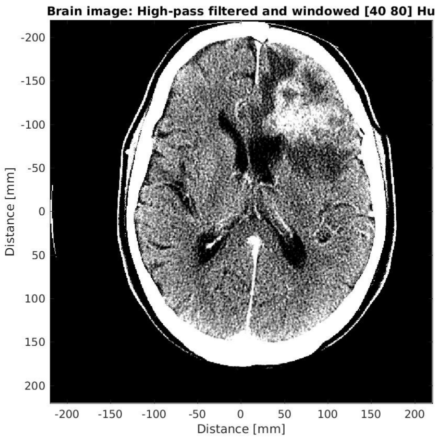
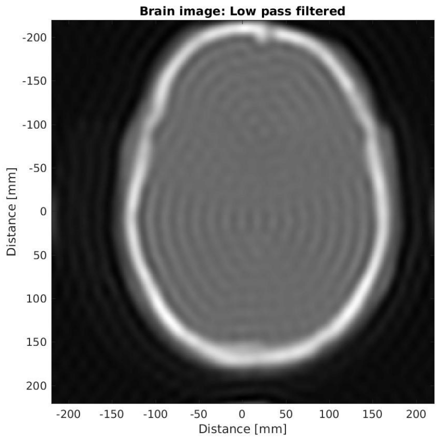




Low-pass Filter







Reconstruction summary

- Filtered backprojection algorithm and choices
- Fan-beam geometry and reconstruction
- Overview of other reconstruction methods –
  - MR, PET, PET/CT
- Advise for the assignments

- Next time: PET imaging by Chief physicist Søren Holm, Clinical Physiology and Nuclear Medicine, Rigshospitalet
- Later Algebraic reconstruction with Senior researcher, PhD Jakob Sauer Jørgensen, DTU Compute
- Reading material:
  - Prince & Links chapter 6-9, 12 & 13