


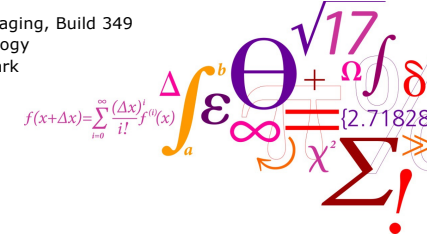
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Reconstruction in CT and hints to the assignments

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$$f(x+\Delta x) = \sum_{i=0}^{\infty} \frac{(\Delta x)^i}{i!} f^{(i)}(x)$$


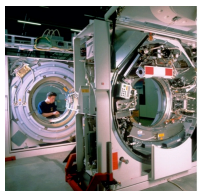
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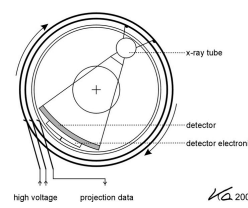
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CT reconstruction repetition & hints

- Filtered backprojection algorithm
 - Radon transform
 - Filtered backprojection
 - Filters and their impulse responses




- Advise for the assignments
- Reading material: Prince & Links Chapter 6



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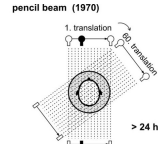
From: W. A. Kalender; Computed Tomography, Publicis, 2005

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Modern CT system generations

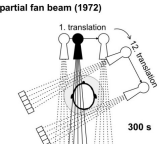
pencil beam (1970)



> 24 h

1st generation: translation / rotation

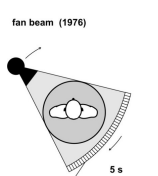
partial fan beam (1972)



300 s

2nd generation: translation / rotation

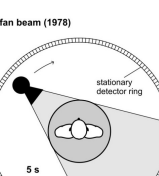
fan beam (1976)



5 s

3rd generation: continuous rotation

fan beam (1978)




5 s

4th generation: continuous rotation

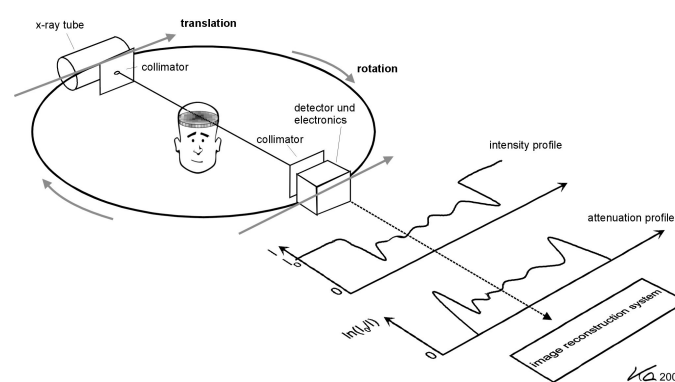
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From: W. A. Kalender; Computed Tomography, Publicis, 2005

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Measurement of attenuation



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Parallel beam projection

1. translation
60. translation
> 24 h

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From: W. A. Kalender; Computed Tomography, Publicis, 2005

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Parallel beam projection geometry

CT coordinate system
Patient coordinate system

Point
 ψ
 θ
 ϕ

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Radon transform

$$p(x', \phi) = \int_{-\infty}^{+\infty} f(x' \cos \phi - y' \sin \phi, x' \sin \phi + y' \cos \phi) dy'$$

$f(x, y)$ – Attenuation image
 x', y' – Gantry coordinate system
 x, y – Patient coordinate system

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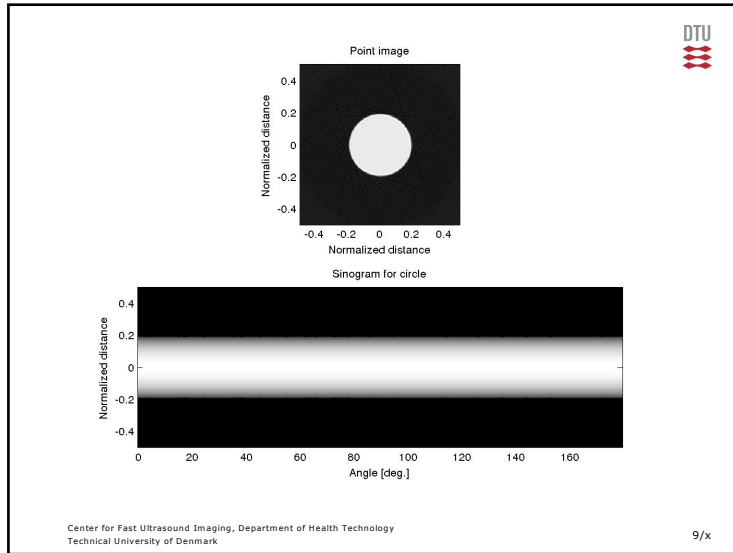
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Point image

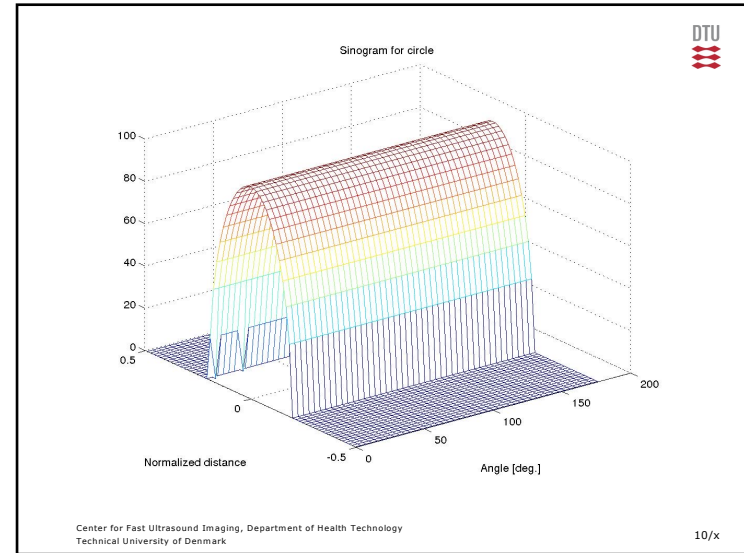
Normalized distance
Normalized distance
Angle [deg.]

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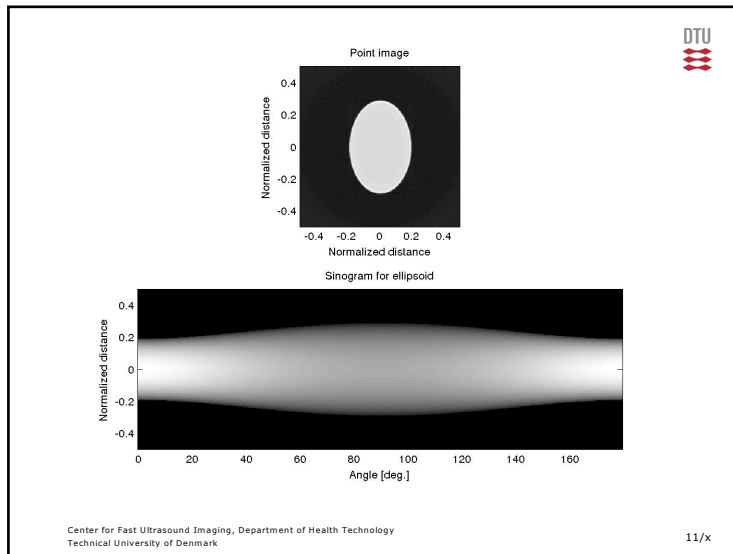
8



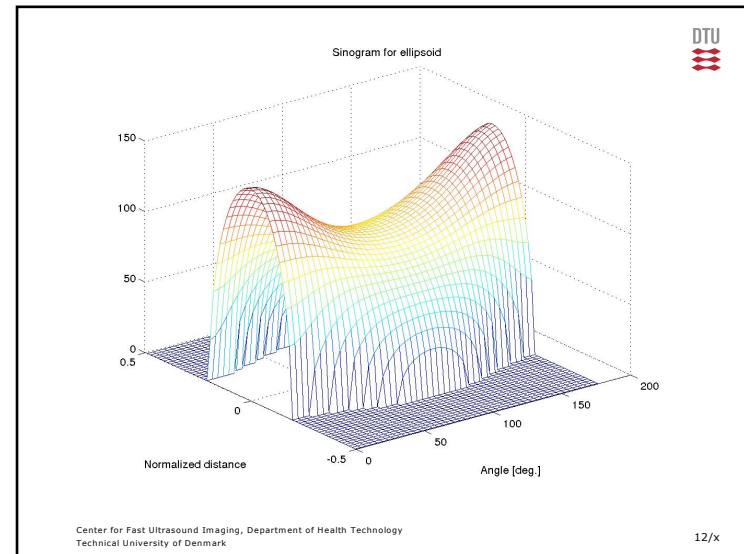
9



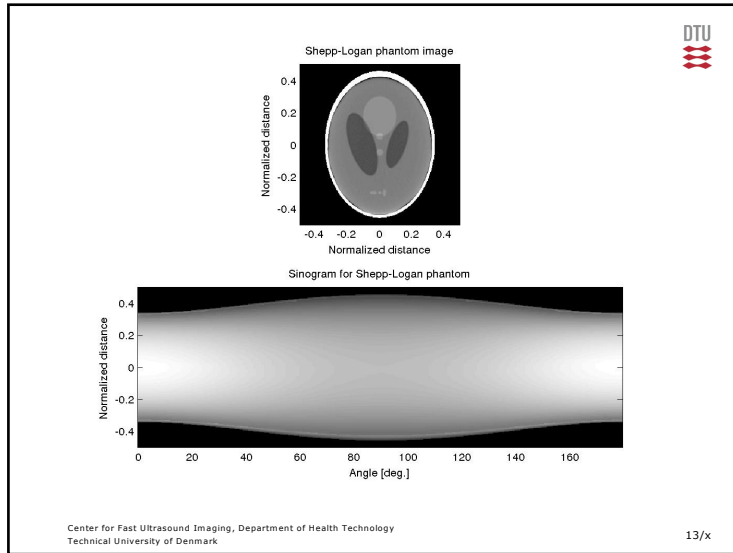
10



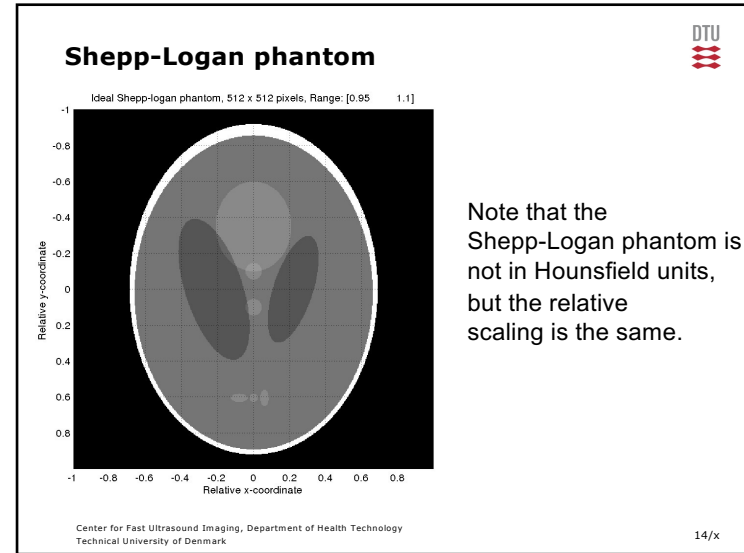
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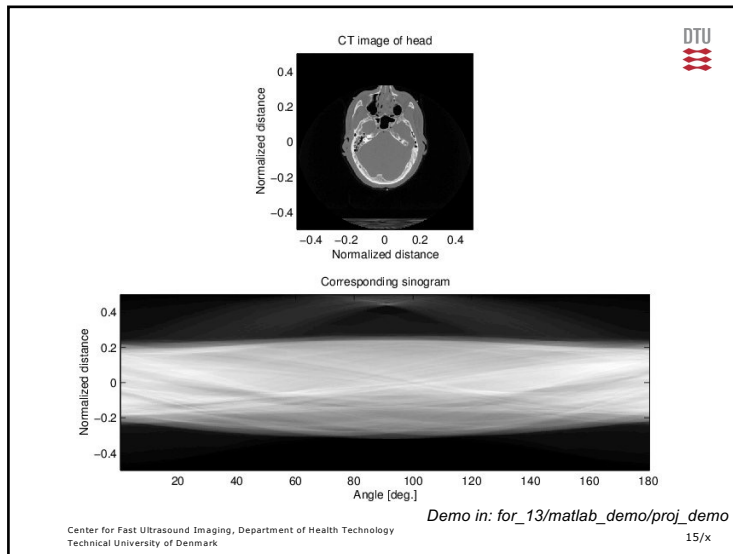
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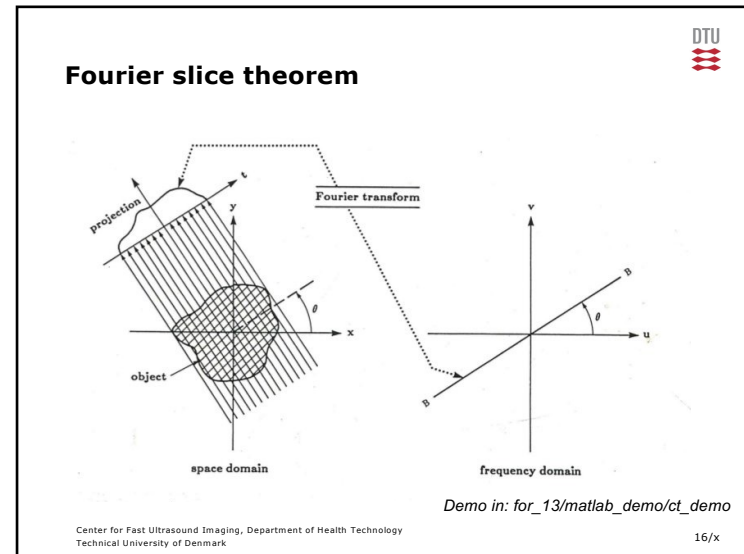
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Fourier Slice Theorem

$$P(\rho, \phi) = \int_{-\infty}^{+\infty} p(x', \phi) e^{-j2\pi\rho x'} dx'$$

$$= F(\rho \cos \phi, \rho \sin \phi)$$

$F(u, v)$ – 2D Fourier transform of image
 ϕ – Gantry rotation

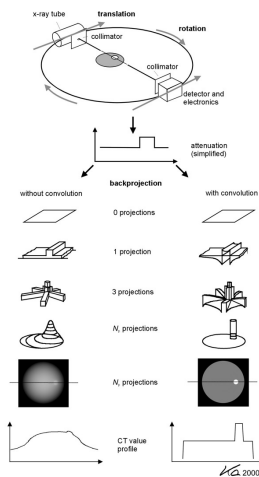
Filtered Back Projection (FBP)

$$\hat{f}(x, y) = \int_0^{\pi} \int_{-\infty}^{+\infty} |\rho| P(\rho, \phi) e^{j2\pi\rho x'} d\rho d\phi$$

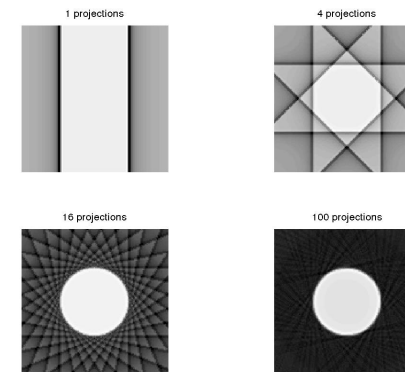
$\hat{f}(x, y)$ – Reconstructed image
 ϕ – Gantry rotation
 x' – Detector position


Filtered backprojection

- Perform for all projections:
 - Make Fourier transform of projected data
 - Apply filter in Fourier domain
 - Make invers transform
 - Back-project and sum with previous image



Influence from number of projections

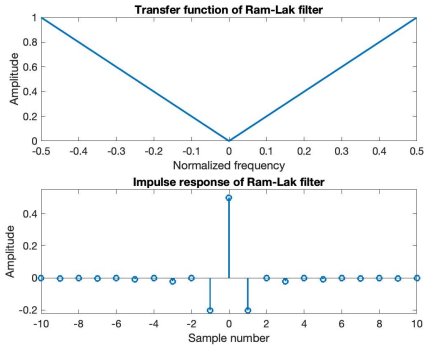


Ram-Lak filter 

Transfer function:


$$h(\rho) = \begin{cases} |\rho|, & |\rho| \leq B \\ 0 & \text{else} \end{cases}$$

Impulse response

$$h(k) = \begin{cases} B^2 & k = 0 \\ -\frac{B^2}{\left(\frac{\pi}{2}k\right)^2} & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$$


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Impulse response for Ram-Lak filter 

- Impulse response of square wave:


$$h_{\text{square}}(x) = A2B \frac{\sin 2\pi B x}{2\pi B x}$$

- Ram-Lak impulse response: square wave – triangle:

$$h_{\text{ramlak}}(x) = 2BB \frac{\sin 2\pi B x}{2\pi B x} - B^2 \frac{\sin^2 \pi B x}{(\pi B x)^2}$$

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Sampling of impulse response for Ram-Lak filter 

- Sampling at $\frac{1}{\Delta x} = 2B, x = k \Delta x = \frac{k}{2B}$:

$$h_{\text{ramlak}}(k) = 2B^2 \frac{\sin 2\pi B \frac{k}{2B}}{2\pi B \frac{k}{2B}} - B^2 \frac{\sin^2 \pi B \frac{k}{2B}}{(\pi B \frac{k}{2B})^2}$$

$$\sin 2\pi B \frac{k}{2B} = \sin \pi k = 0$$

$$\sin^2 \pi B \frac{k}{2B} = \sin^2 \frac{\pi}{2} k = 1 \text{ (k odd) else } 0$$

- Gives:


$$h(0) = B^2$$

$$h(k) = -B^2 \frac{\sin^2 k \frac{\pi}{2}}{\left(k \frac{\pi}{2}\right)^2} = -\frac{B^2}{\left(k \frac{\pi}{2}\right)^2} \text{ (k odd)}$$

$$h(k) = 0 \text{ else}$$

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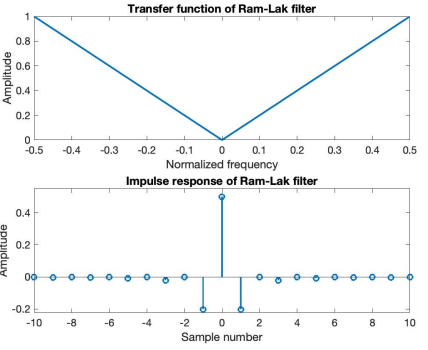
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Ram-Lak filter 

Transfer function:

$$h(\rho) = \begin{cases} |\rho|, & |\rho| \leq B \\ 0 & \text{else} \end{cases}$$

Impulse response

$$h(k) = \begin{cases} B^2 & k = 0 \\ -\frac{B^2}{\left(\frac{\pi}{2}k\right)^2} & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$$


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Transfer function of filters – Ram-Lak

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Shepp-Logan filter

Transfer function:

$$h(\rho) = \begin{cases} \left| \rho \frac{\sin \frac{2\pi\rho}{4B}}{\frac{2\pi\rho}{4B}} \right|, & |\rho| \leq B \\ 0 & \text{else} \end{cases}$$

Impulse response

$$h(k) = -\frac{8B^2}{\pi(4k^2 - 1)}$$

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Shepp-Logan filter

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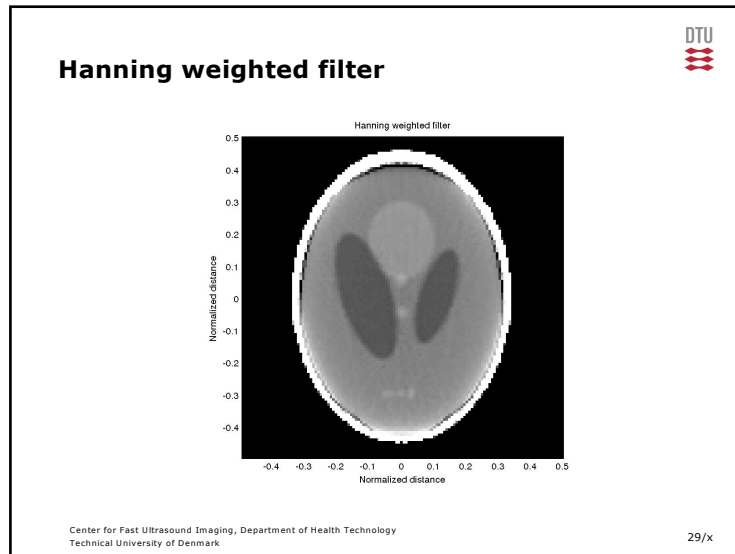
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Hanning weighted filter

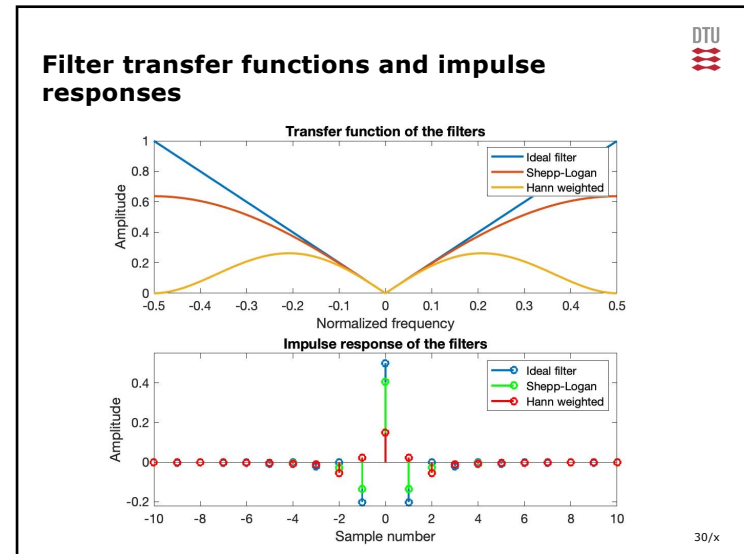
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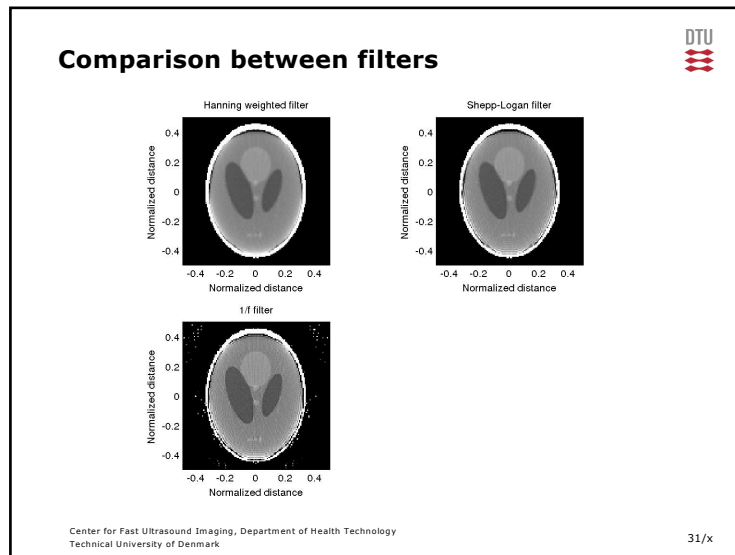
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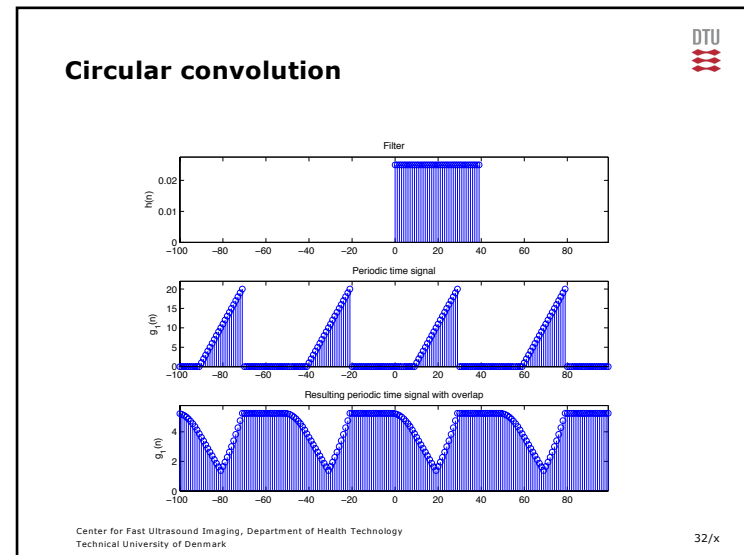
29



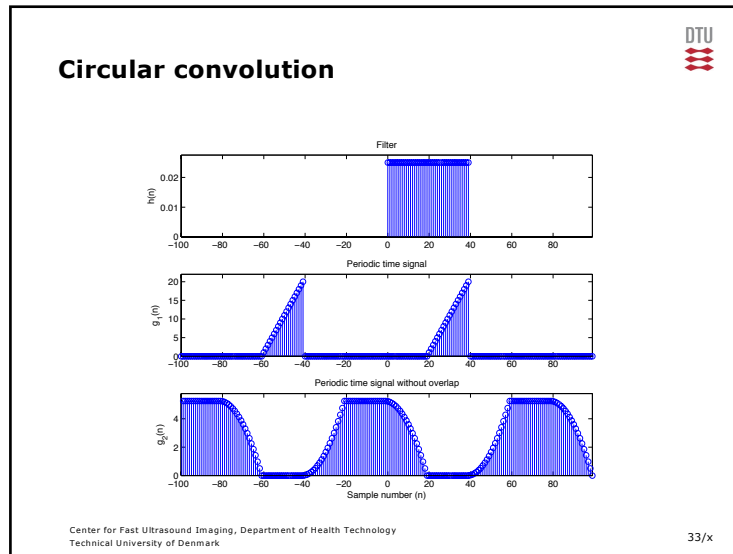
30



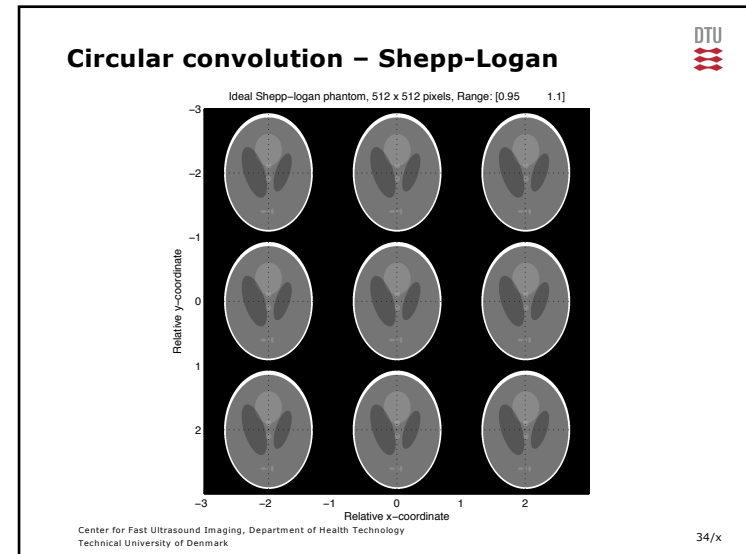
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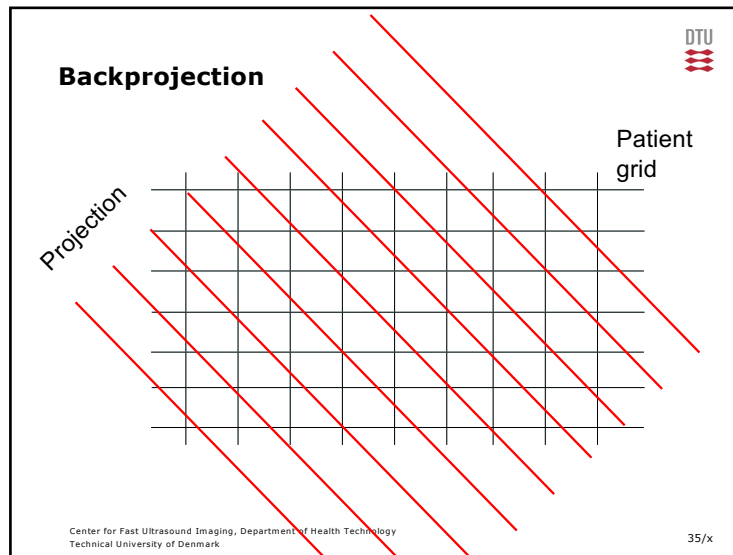
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Data for testing and validation

- Use data sets on web site
- Circular phantom for geometry test
- Shepp-Logan for orientation and quantitative data
- In-vivo images for Hounsfield units

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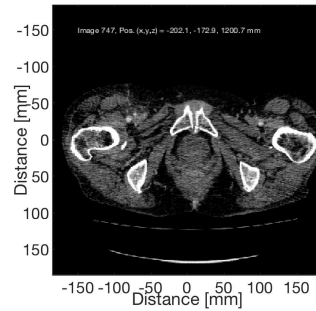
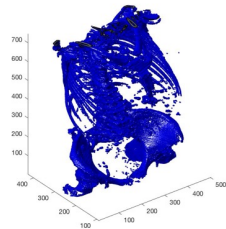
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Assignment data

- DICOM data from female patient
- All data available on the web
- Task is to find which slice it is



Data and program in:
[undervisning/K_22485_31545_billeder/ct_data/dicom_data](#)

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Reading DICOM data

```
% Set overall parameters
dir_name='DICOM/ST00001/SE00001/'; % Directory name
start_image=1; % First image in series
end_image=747; % Last image in series
frame_rate=50; % Frame rate for playing back the movie

% Set the dynamic range for the display
off_set=100; % Offset [Hu]
range=400; % Range to display [Hu]
map_values=128; % Number of gray level values
bone_off_set=-250; % Offset for showing the bones
bone_range=100; % Range for showing the bones

% Initialize figure
colormap(gray(map_values));
dicom_movie(end_image+1-start_image) = struct('cdata', [], 'colormap', []);

% Read information for the first images
file_name='IM0001';
info=dicominfo([dir_name, file_name]);
dx=info.PixelSpacing(1);
dy=info.PixelSpacing(2);
Y = dicomread(info);
[Nx,Ny]=size(Y);

% Make space for all the images
Y=zeros(Nx,Ny,end_image+1-start_image);
z_positions=zeros(end_image+1-start_image,1);
```

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```
% Loop through the images and read and display them
for i=start_image:end_image
file_name=[ 'IM00',num2str(floor(i/100)),num2str(floor(rem(i,100)/10)),num2str(rem(i,10))];
info=dicominfo([dir_name, file_name]);
Y(:, :, i) = dicomread(info);
image((1:Nx)-Nx/2)*dx, ((1:Ny)-Ny/2)*dy, (double(Y(:, :, i))+off_set)/range*map_values)
xlabel('Distance [mm]')
ylabel('Distance [mm]')
pos=sprintf('%5.1f, %5.1f, %5.1f', info.ImagePositionPatient(1), ...
info.ImagePositionPatient(2), info.ImagePositionPatient(3));
z_positions(i)= info.ImagePositionPatient(3);
text(-150, -150, ['Image ', num2str(i), ', Pos. (x,y,z) = ', pos, ' mm'], 'Color', [1 1 1])
axis('image')
drawnow
dicom_movie(i)=getframe;
end

% Display the movie
movie(dicom_movie, 5, frame_rate);
```

Full script can be found at:

courses.healthtech.dtu.dk/22485/files/ct_data/dicom_data/display_dicom_images.m

on the page for the CT data: courses.healthtech.dtu.dk/22485/?ct_data/assign_data.html

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Script in: ct_data/dicom_data 39/x

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