Reconstruction in CT and hints to the assignments

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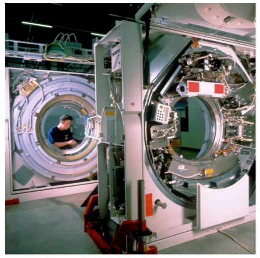
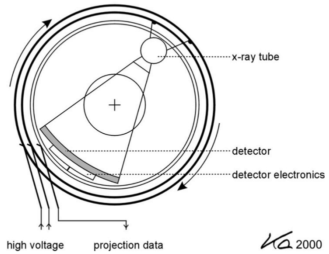
CT reconstruction repetition & hints

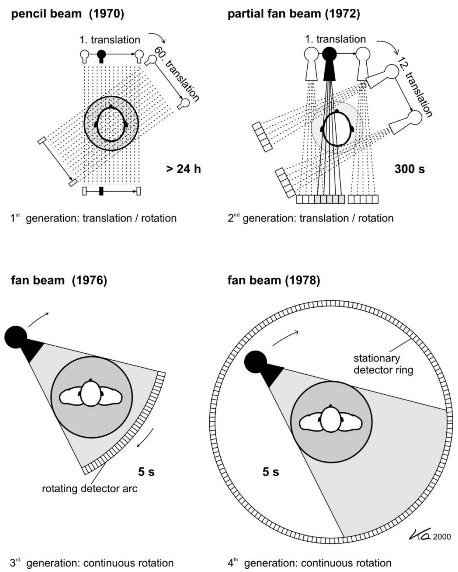
- Filtered backprojection algorithm
  - Radon transform
  - Filtered backprojection
  - Filters and their impulse responses
- Advise for the assignments
- Reading material: Prince & Links Chapter 6





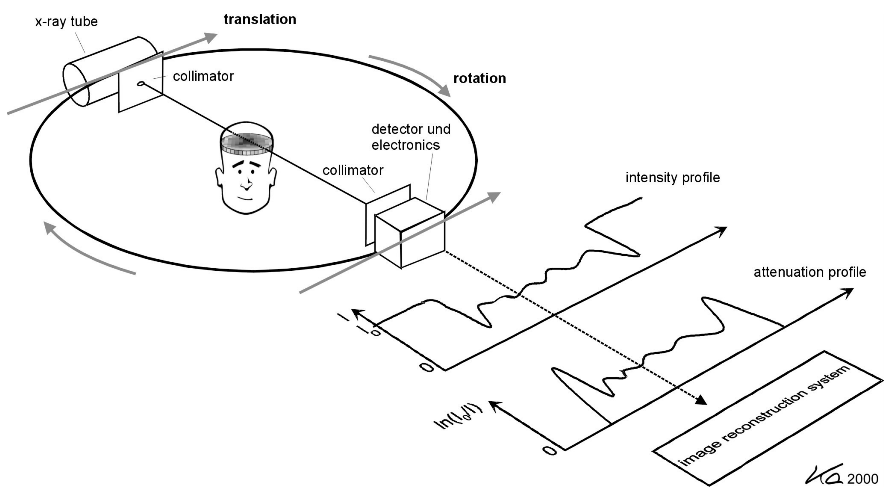
From: W. A. Kalender; Computed Tomography, Publicis, 2005

Modern CT system generations



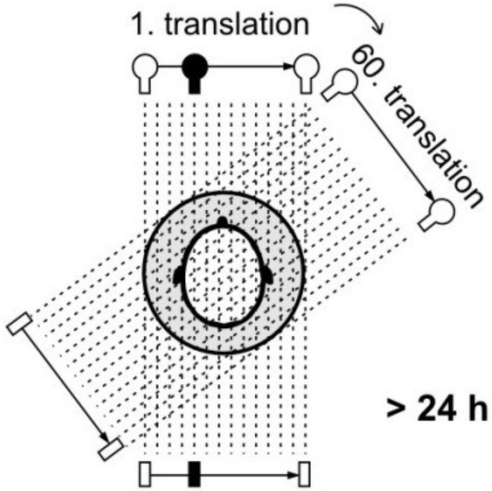
From: W. A. Kalender; Computed Tomography, Publicis, 2005

Measurement of attenuation



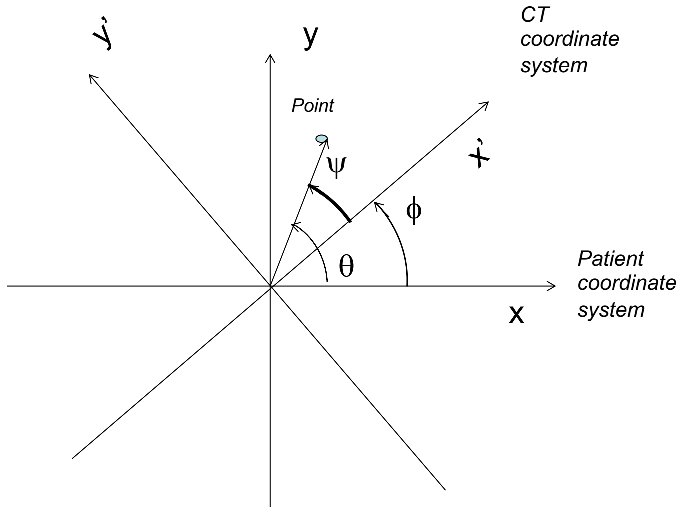
From: W. A. Kalender; Computer Tomography, Publicis, 2005

Parallel beam projection



From: W. A. Kalender; Computed Tomography, Publicis, 2005

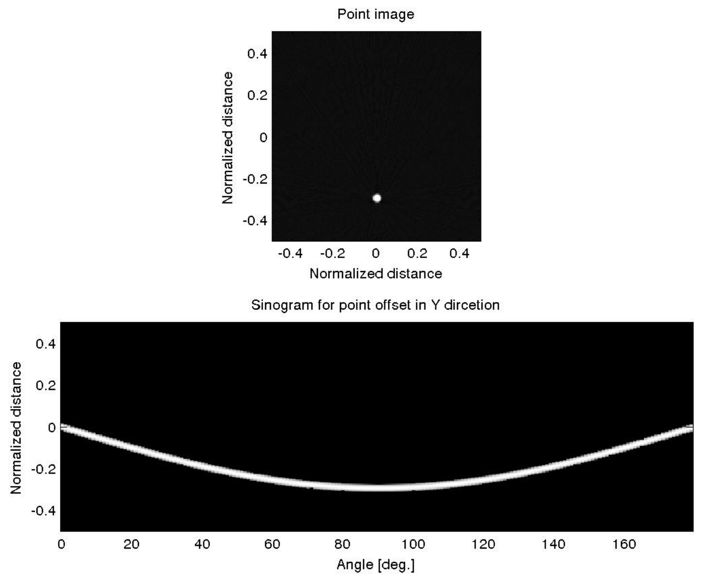
Parallel beam projection geometry

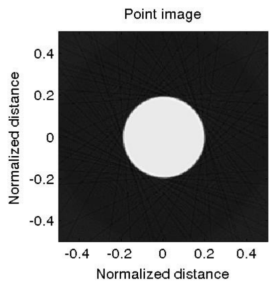


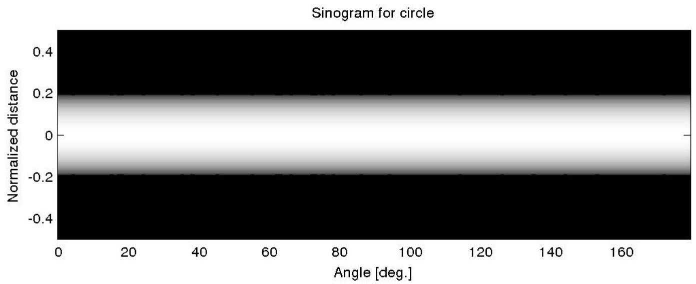
Radon transform

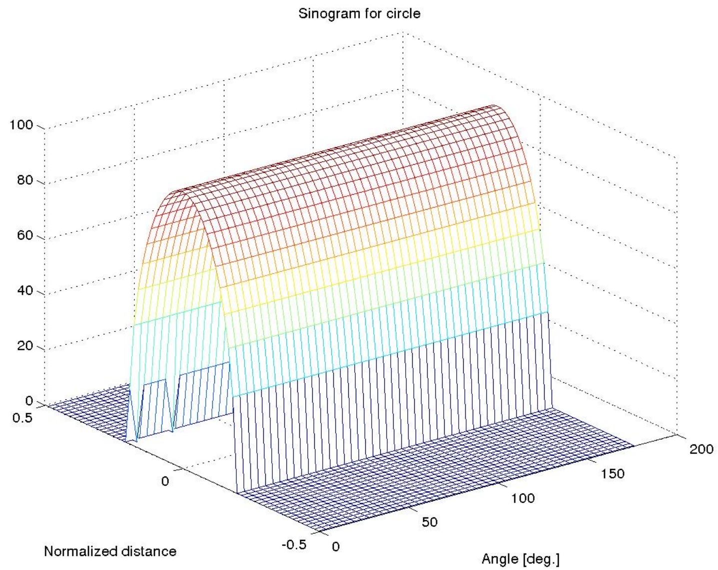
$$p(x',\phi) = \int_{-\infty}^{+\infty} f(x'\cos\phi - y'\sin\phi, x'\sin\phi + y'\cos\phi)\,dy'$$

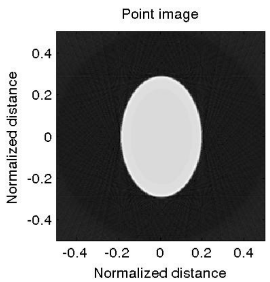
f(x,y) – Attenuation image
x', y' – Gantry coordinate system
x, y – Patient coordinate system

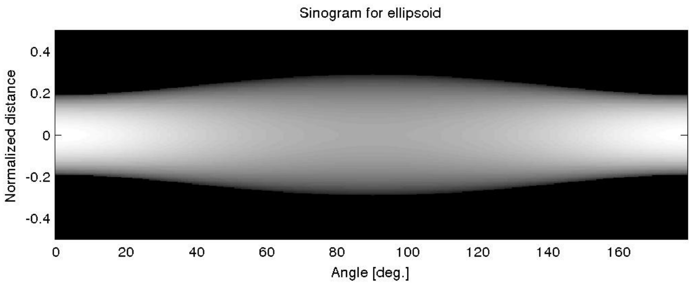


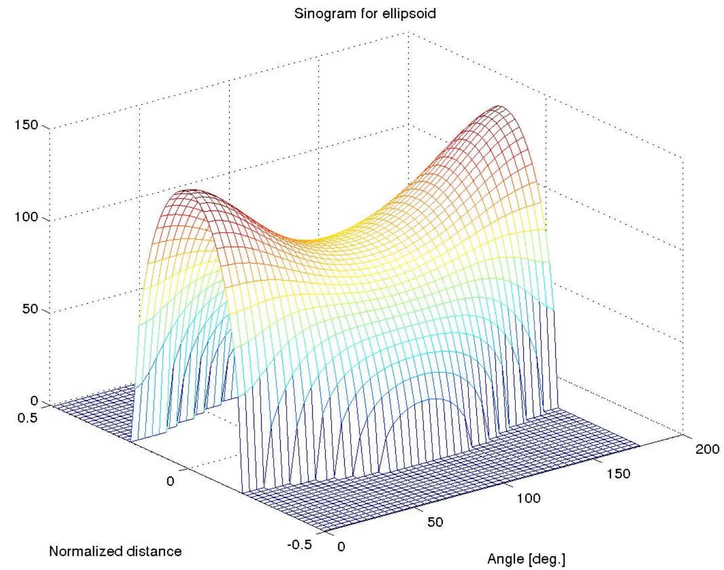

Point image
Normalized distance
Normalized distance

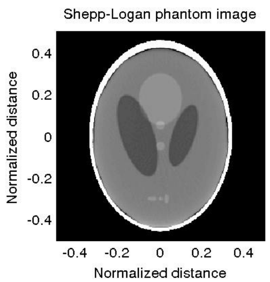

Sinogram for circle
Normalized distance
Angle [deg.]

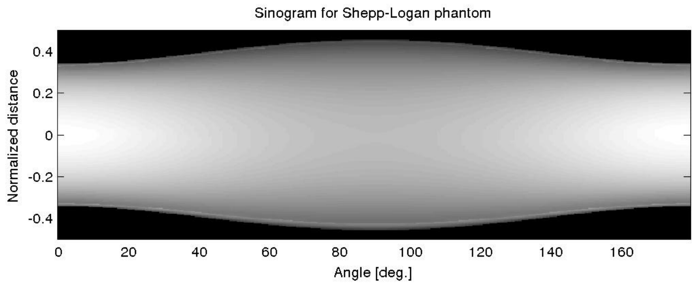

Sinogram for circle
Normalized distance
Angle [deg.]


Point image
Normalized distance
0.4
0.2
0
-0.2
-0.4
-0.4 -0.2 0 0.2 0.4
Normalized distance

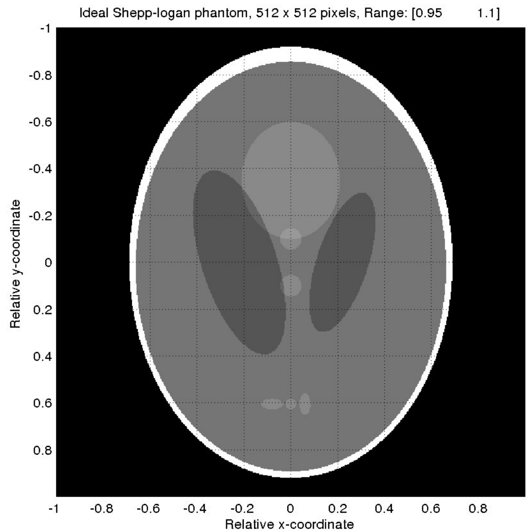

Sinogram for ellipsoid
Normalized distance
0.4
0.2
0
-0.2
-0.4
0 20 40 60 80 100 120 140 160
Angle [deg.]


Sinogram for ellipsoid
150
100
50
0
0.5
0
-0.5
Normalized distance
0
50
100
150
200
Angle [deg.]

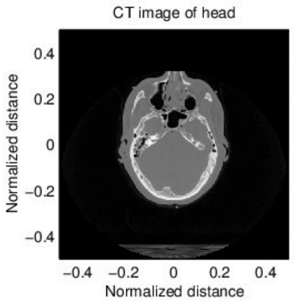


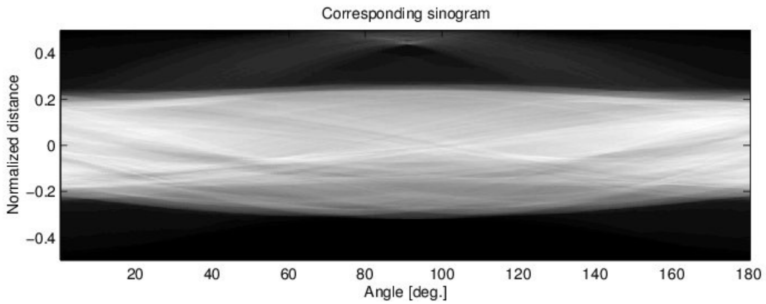


Shepp-Logan phantom



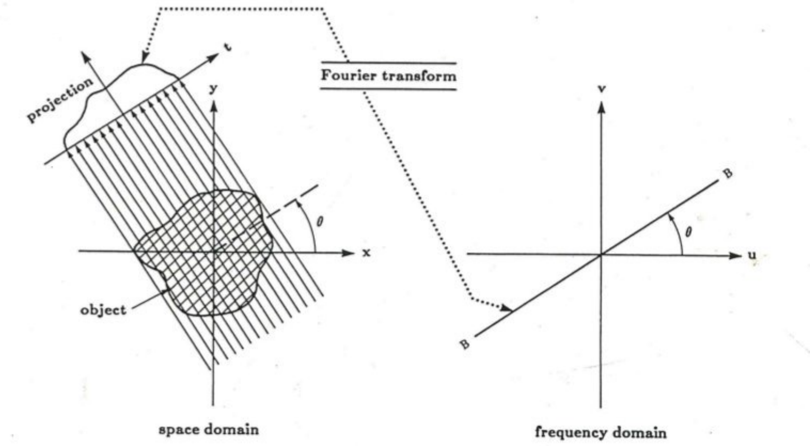
Note that the Shepp-Logan phantom is not in Hounsfield units, but the relative scaling is the same.





Demo in: for_13/matlab_demo/proj_demo

Fourier slice theorem



Demo in: for_13/matlab_demo/ct_demo

Fourier Slice Theorem

$$P(\rho,\phi) = \int_{-\infty}^{+\infty} p(x',\phi) e^{-j2\pi\rho x'} dx'$$

$$= F(\rho\cos\phi, \rho\sin\phi)$$

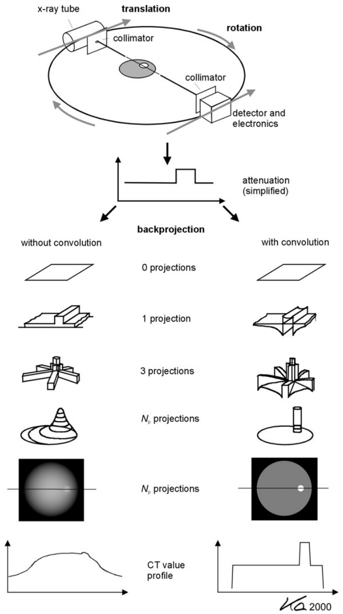
F(u,v) – 2D Fourier transform of image
ϕ – Gantry rotation

Filtered Back Projection (FBP)

$$\hat{f}(\mathrm{x},\mathrm{y}) = \int_{0}^{\pi} \int_{-\infty}^{+\infty} |\rho| P(\rho,\phi) e^{j2\pi\rho x'} d\rho d\phi$$

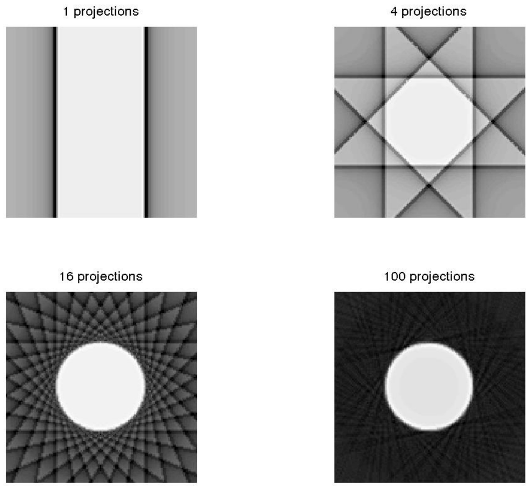
$\hat{f}(\mathrm{x},\mathrm{y})$ – Reconstructed image
ϕ – Gantry rotation
x' – Detector position

Filtered backprojection



- Perform for all projections:
  - Make Fourier transform of projected data
  - Apply filter in Fourier domain
  - Make invers transform
  - Back-project and sum with previous image

Influence from number of projections



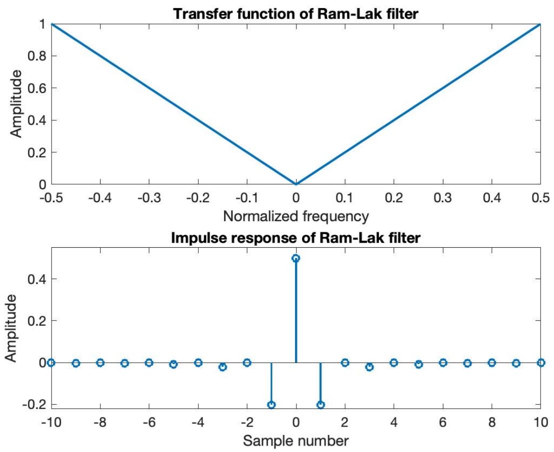
Ram-Lak filter

Transfer function:

$$h(\rho) = \begin{cases} |\rho|, & |\rho| \leq B \\ 0 & else \end{cases}$$

Impulse response

$$h(k) = \begin{cases} B^2 & k = 0 \\ -\dfrac{B^2}{\left(\dfrac{\pi}{2}k\right)^2} & k\ odd \\ 0 & k\ even \end{cases}$$



Impulse response for Ram-Lak filter

- Impulse response of square wave:

$$h_{square}(x) = A2B\,\frac{\sin 2\pi B\,x}{2\pi Bx}$$

- Ram-Lak impulse response: square wave – triangle:

$$h_{ramlak}(x) = 2BB\,\frac{\sin 2\pi Bx}{2\pi Bx} - B^2\,\frac{\sin^2 \pi Bx}{(\pi Bx)^2}$$

Sampling of impulse response for Ram-Lak filter

- Sampling at $\frac{1}{\Delta x} = 2B, x = k\ \Delta x = \frac{k}{2B}$:

$$h_{ramlak}(k) = 2B^2 \frac{\sin 2\pi B \frac{k}{2B}}{2\pi B \frac{k}{2B}} - B^2 \frac{\sin^2 \pi B \frac{k}{2B}}{(\pi B \frac{k}{2B})^2}$$

$$\sin 2\pi B \frac{k}{2B} = \sin \pi k = 0$$

$$\sin^2 \pi B \frac{k}{2B} = \sin^2 \frac{\pi}{2} k = 1\ (k\ odd) else\ 0$$

- Gives:

$$h(0) = B^2$$

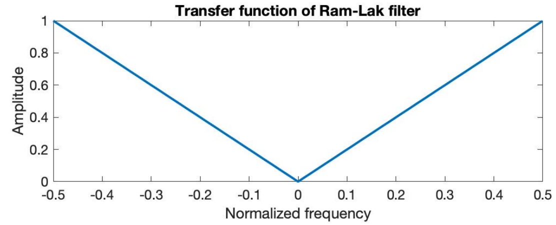
$$h(k) = -B^2 \frac{\sin^2 k\frac{\pi}{2}}{(k\frac{\pi}{2})^2} = -\frac{B^2}{(k\frac{\pi}{2})^2} \text{ (k odd)}$$

$$h(k) = 0 \text{ else}$$

Ram-Lak filter

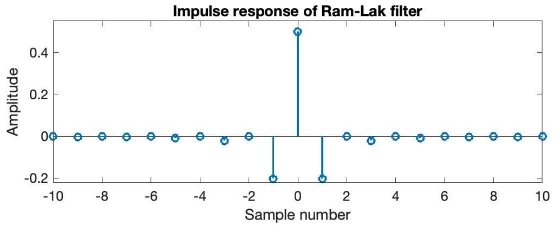
Transfer function:

$$h(\rho) = \begin{cases} |\rho|, & |\rho| \le B \\ 0 & else \end{cases}$$

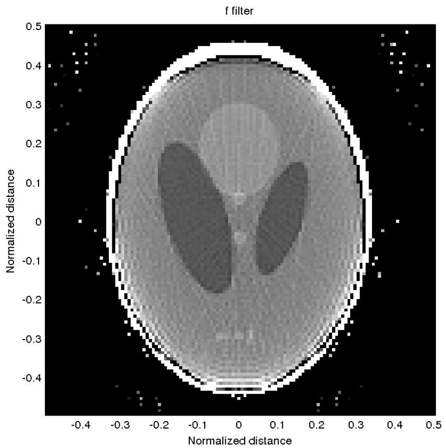


Impulse response

$$h(k) = \begin{cases} B^2 & k = 0 \\ -\frac{B^2}{\left(\frac{\pi}{2}k\right)^2} & k\ odd \\ 0 & k\ even \end{cases}$$



Transfer function of filters – Ram-Lak



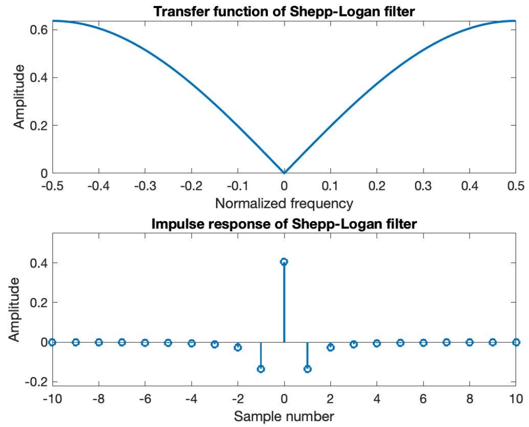
Shepp-Logan filter

Transfer function:

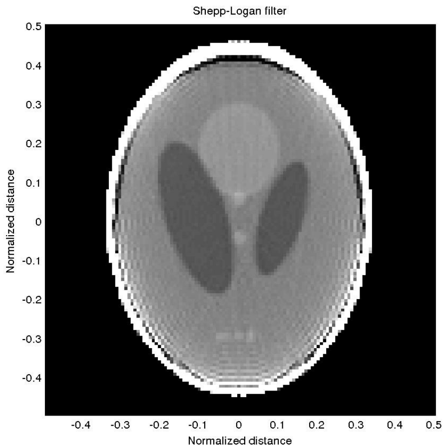
$$h(\rho) = \begin{cases} |\rho| \dfrac{\sin \frac{2\pi\rho}{4B}}{\frac{2\pi\rho}{4B}}, |\rho| \le B \\ 0 \quad else \end{cases}$$

Impulse response

$$h(k) = -\frac{8B^2}{\pi(4k^2 - 1)}$$



Shepp-Logan filter



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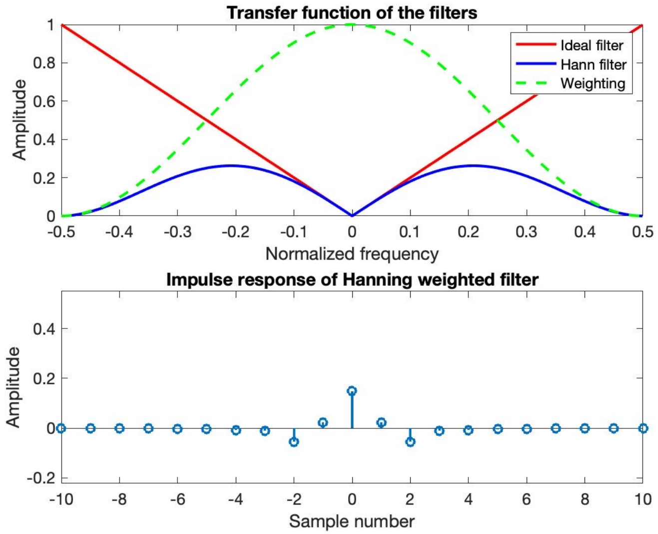
27/x

27

Hanning weighted filter



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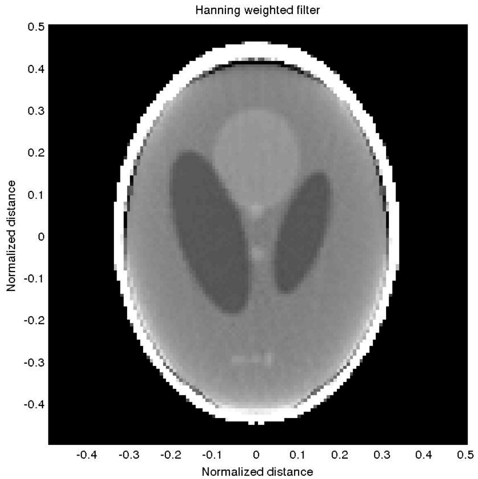
28/x

28

Hanning weighted filter



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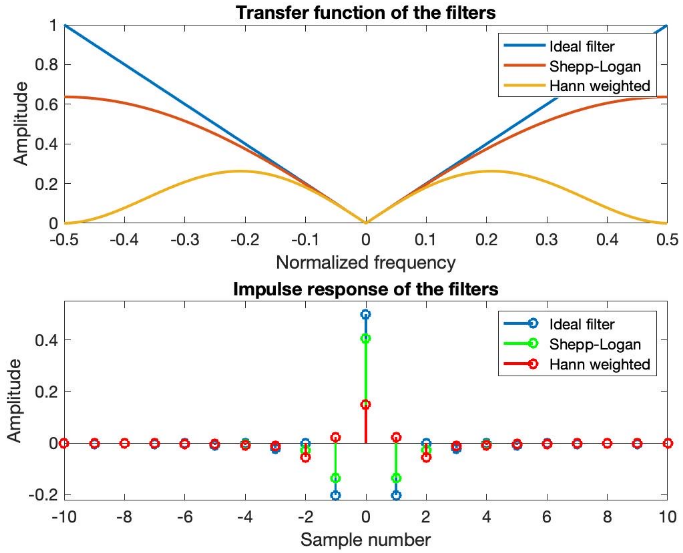
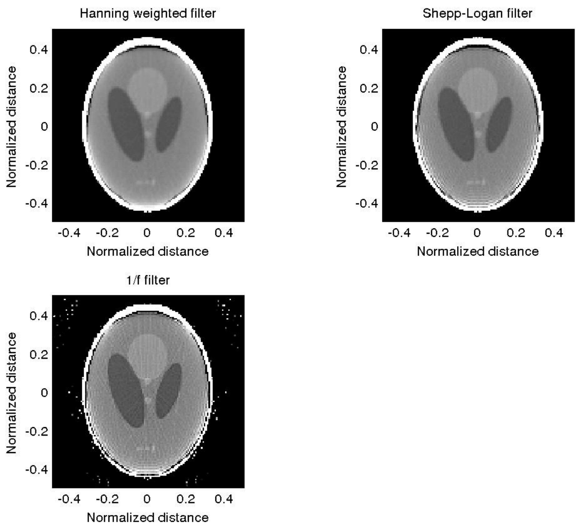
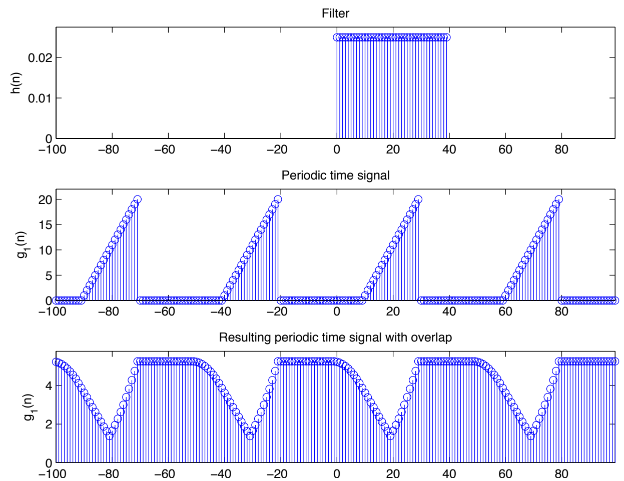
Filter transfer functions and impulse responses



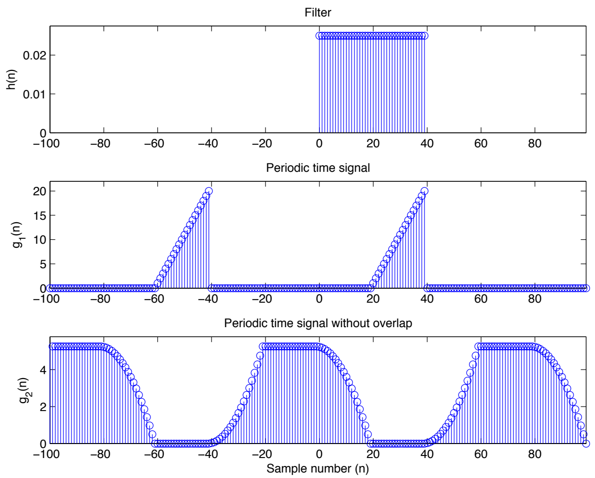
Comparison between filters



Circular convolution



Circular convolution



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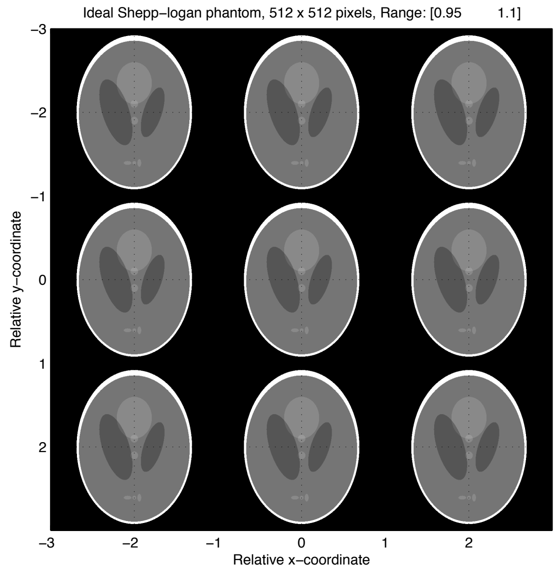
33/x

33

Circular convolution – Shepp-Logan



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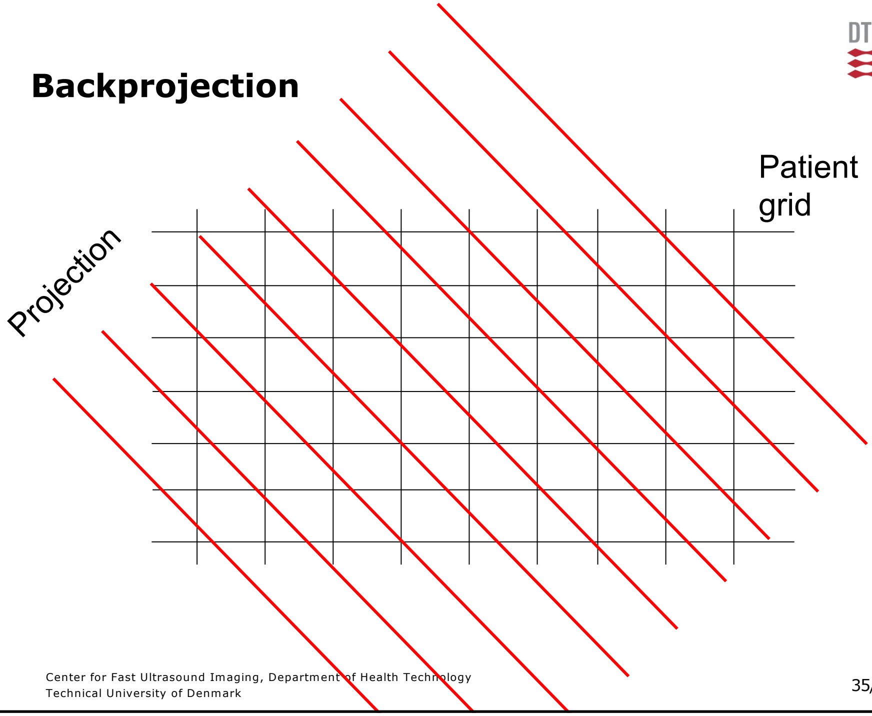
34/x

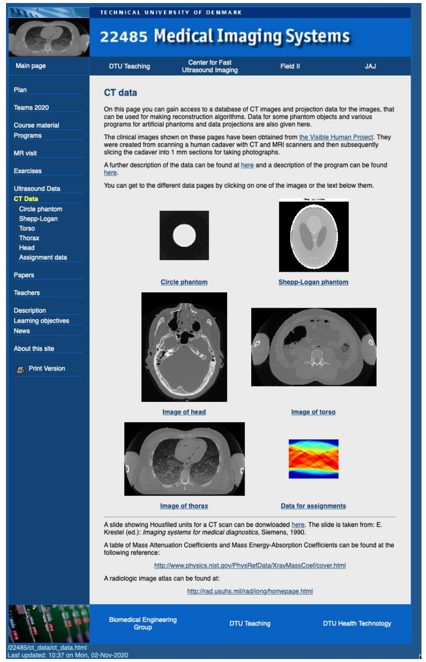
34

Backprojection



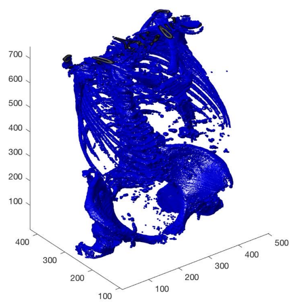
Data for testing and validation

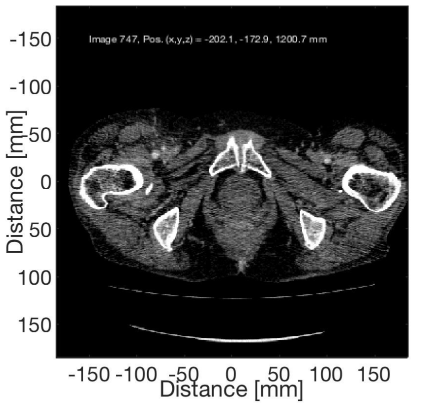
- Use data sets on web site
- Circular phantom for geometry test
- Shepp-Logan for orientation and quantitative data
- In-vivo images for Hounsfield units



Assignment data

- DICOM data from female patient
- All data available on the web
- Task is to find which slice it is





Data and program in:
undervisning/k_22485_31545_billeder/ct_data/dicom_data

Reading DICOM data

```
%  Set overall parameters
dir_name='DICOM/ST00001/SE00001/';  %  Directory name
start_image=1;                      %  First image in series
end_image=747;                      %  Last image in series
frame_rate=50;                      %  Frame rate for playing back the movie

%  Set the dynamic range for the display
off_set=100;          %  Offset [Hu]
range=400;            %  Range to display [Hu]
map_values=128;       %  Number of gray level values
bone_off_set= -250;   %  Offset for showing the bones
bone_range=100;       %  Range for showing the bones

%  Initialize figure
colormap(gray(map_values));
dicom_movie(end_image+1-start_image) = struct('cdata',[],'colormap',[]);

%  Read information for the first images
file_name='IM00001';
info=dicominfo([dir_name, file_name]);
dx=info.PixelSpacing(1);
dy=info.PixelSpacing(2);
Y = dicomread(info);
[Nx,Ny]=size(Y);

%  Make space for all the images
Y=zeros(Nx,Ny,end_image+1-start_image);
z_positions=zeros(end_image+1-start_image,1);
```

```
%  Loop through the images and read and display them

for i=start_image:end_image
    file_name=['IM00',num2str(floor(i/100)),num2str(floor(rem(i,100)/10)),num2str(rem(i,10))];
    info=dicominfo([dir_name, file_name]);
    Y(:,:,i) = dicomread(info);
    image(((1:Nx)-Nx/2)*dx,((1:Ny)-Ny/2)*dy,(double(Y(:,:,i))+off_set)/range*map_values)
    xlabel('Distance [mm]')
    ylabel('Distance [mm]')
    pos=sprintf('%5.1f, %5.1f, %5.1f', info.ImagePositionPatient(1), …
         info.ImagePositionPatient(2), info.ImagePositionPatient(3));
    z_positions(i)= info.ImagePositionPatient(3);
    text(-150, -150, ['Image ', num2str(i),', Pos. (x,y,z) = ', pos,' mm'], 'Color', [1 1 1])
    axis('image')
    drawnow
    dicom_movie(i)=getframe;
end

%  Display the movie

movie(dicom_movie, 5, frame_rate);
```

Full script can be found at:

courses.healthtech.dtu.dk/22485/files/ct_data/dicom_data/display_dicom_images.m

on the page for the CT data: courses.healthtech.dtu.dk/22485/?ct_data/assign_data.html