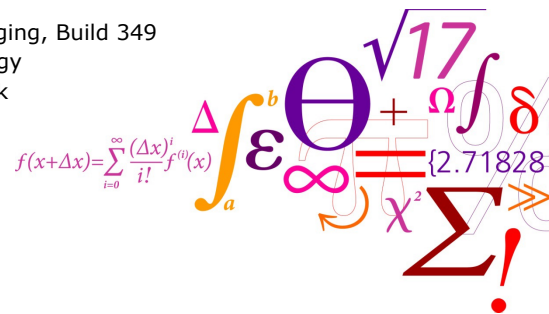


## Reconstruction in CT and hints to the assignments

Jørgen Arendt Jensen

October 26, 2023

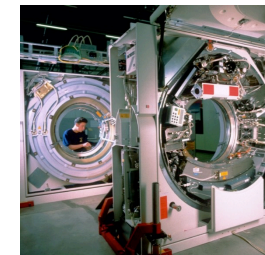
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Department of Health Technology  
Technical University of Denmark



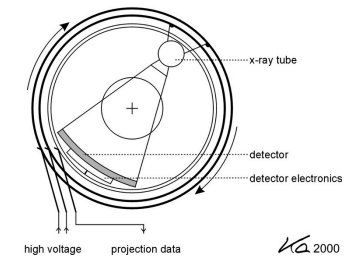
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## CT reconstruction repetition & hints

- Filtered backprojection algorithm
  - Radon transform
  - Filtered backprojection
  - Filters and their impulse responses



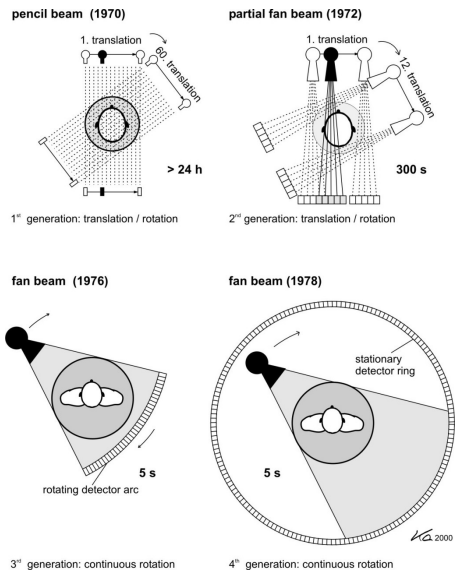
- Advise for the assignments
- Reading material: Prince & Links Chapter 6



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From: W. A. Kalender; Computed Tomography, P.ublicis, 2005

## Modern CT system generations

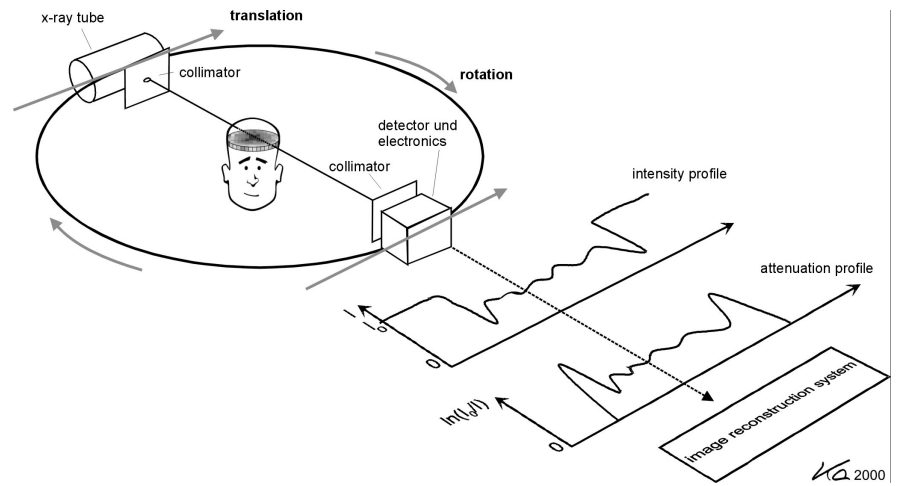


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From: W. A. Kalender; Computed Tomography, Publicis, 2005

## Measurement of attenuation

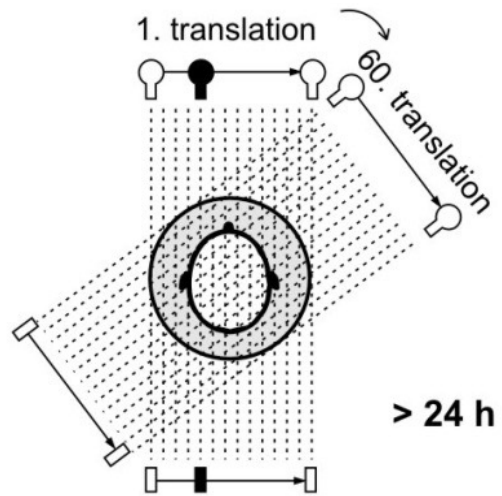


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From: W. A. Kalender; Computer Tomography, Publicis, 2005

## Parallel beam projection



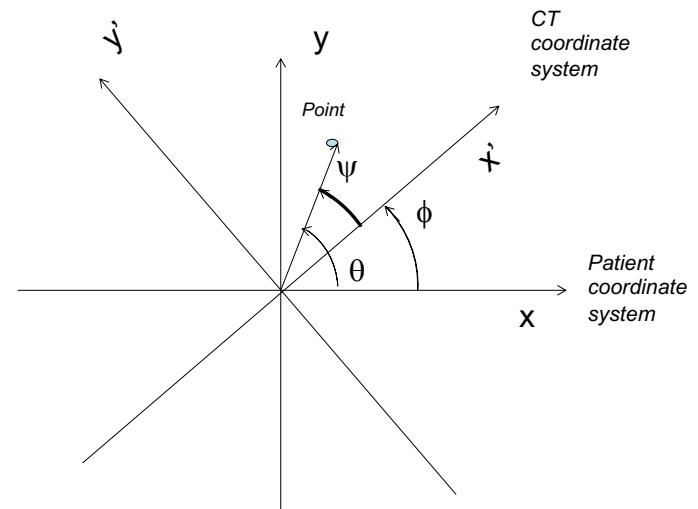
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From: W. A. Kalender; Computed Tomography, Publicis, 2005

5

## Parallel beam projection geometry



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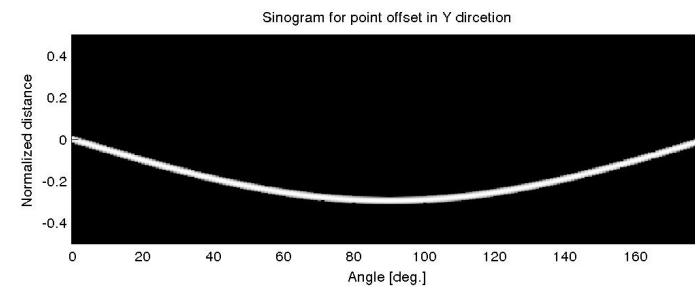
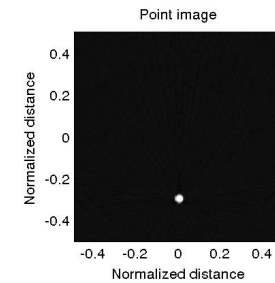
6/x

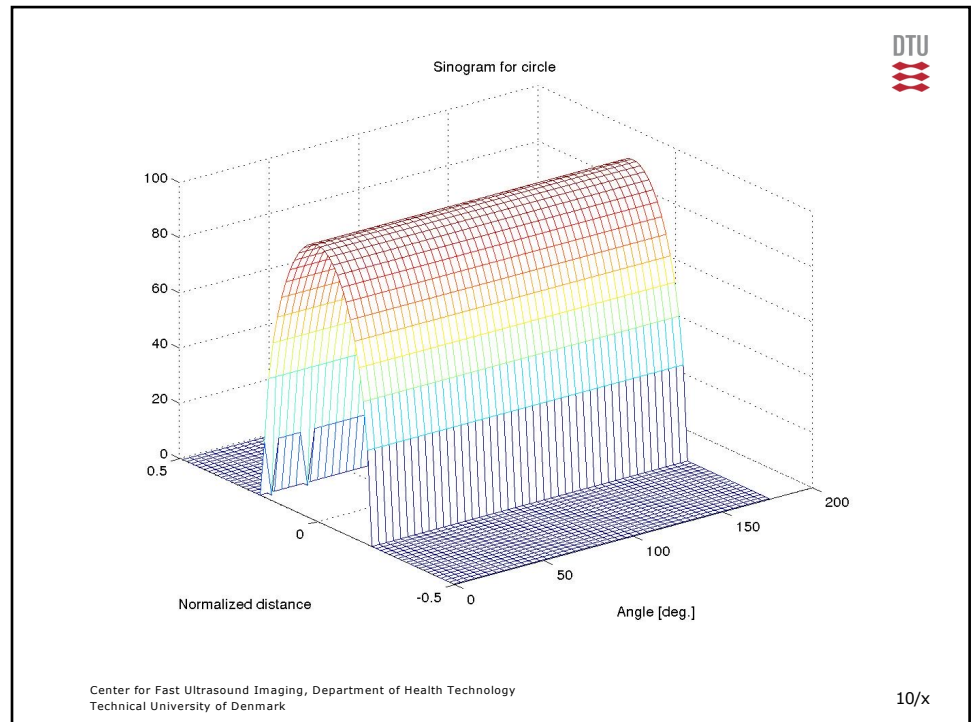
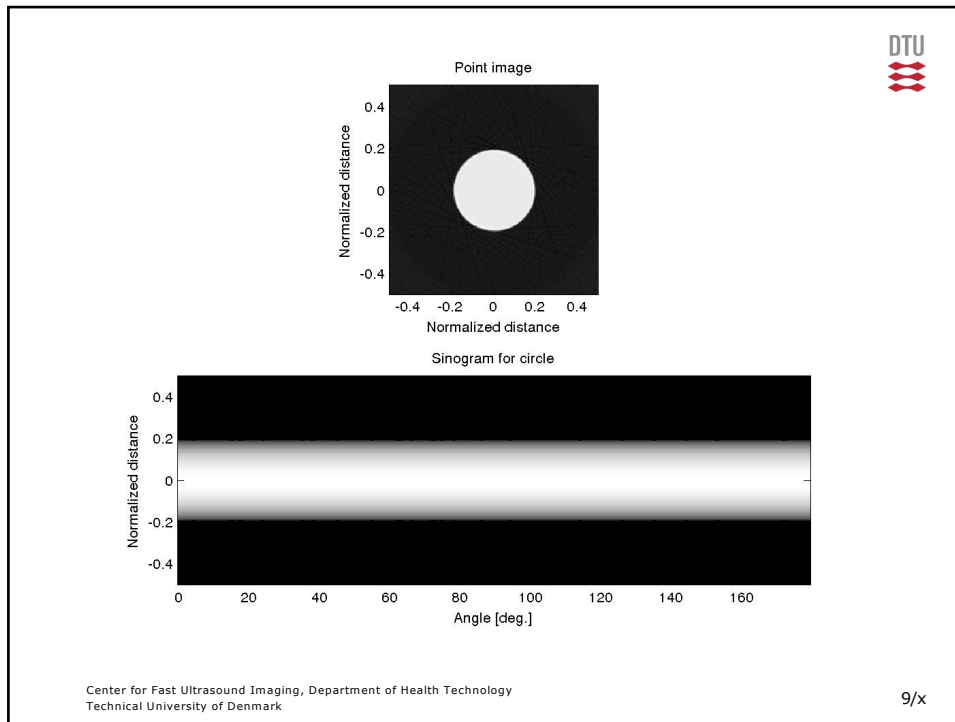
6

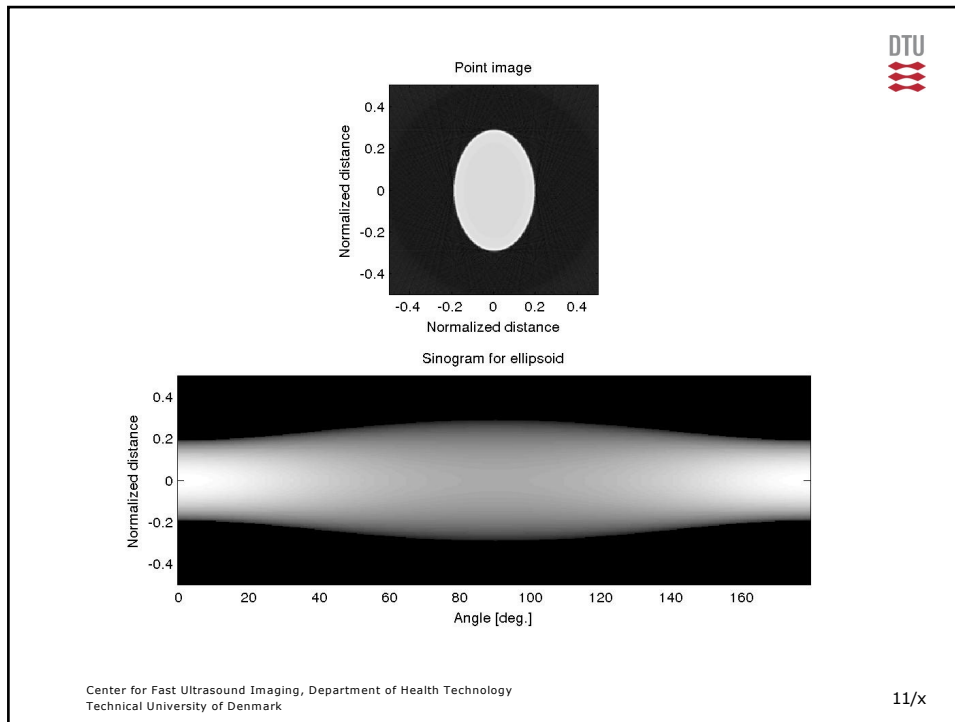
## Radon transform

$$p(x', \phi) = \int_{-\infty}^{+\infty} f(x' \cos \phi - y' \sin \phi, x' \sin \phi + y' \cos \phi) dy'$$

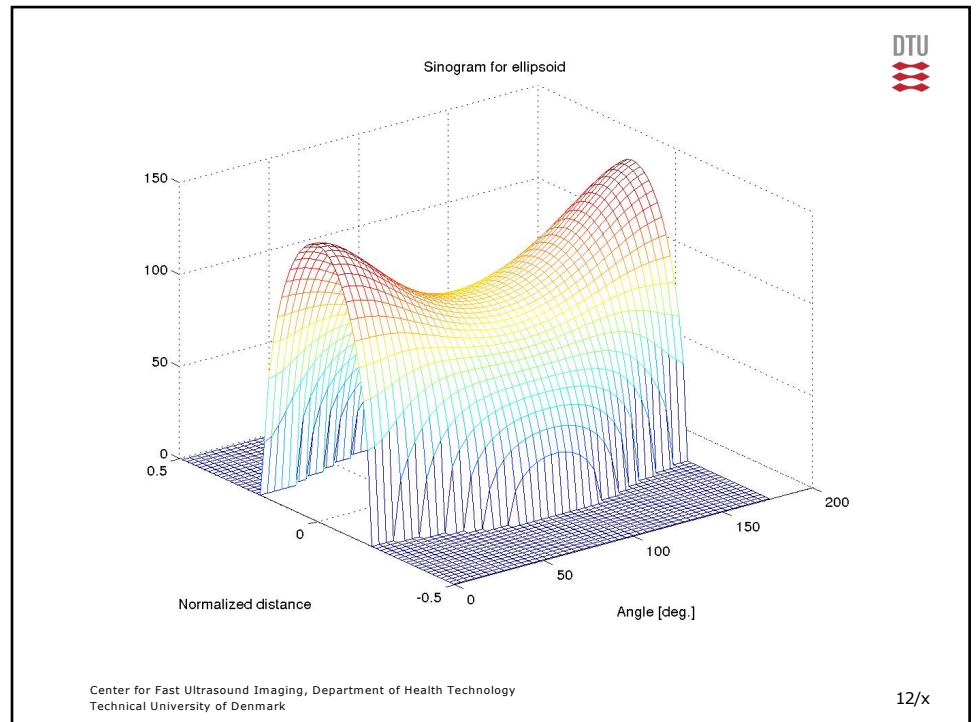
$f(x, y)$  – Attenuation image  
 $x', y'$  – Gantry coordinate system  
 $x, y$  – Patient coordinate system



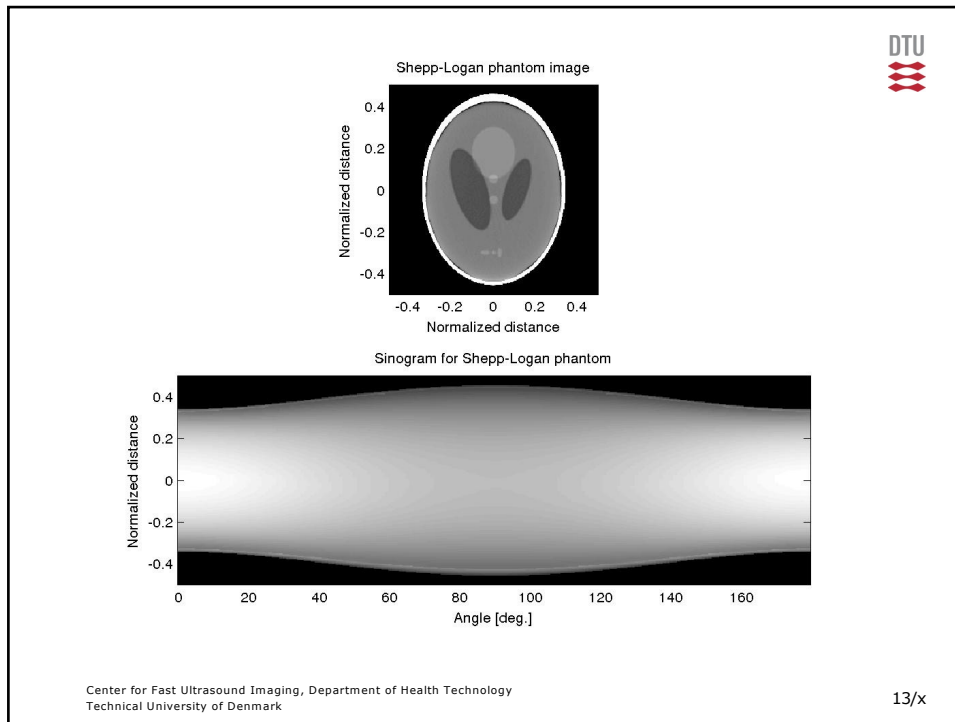




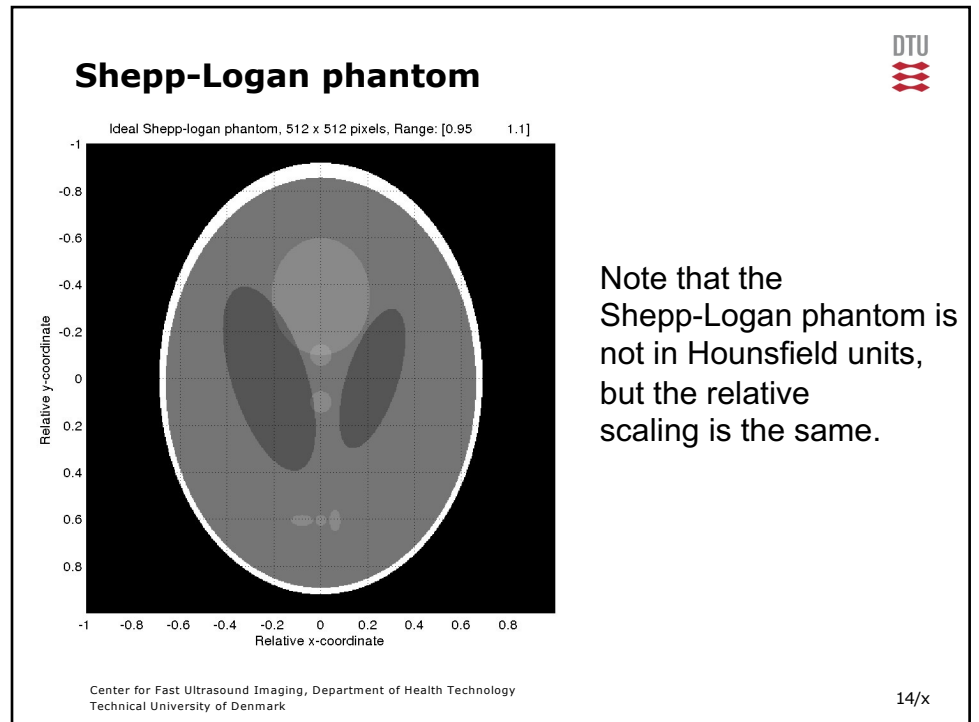
11



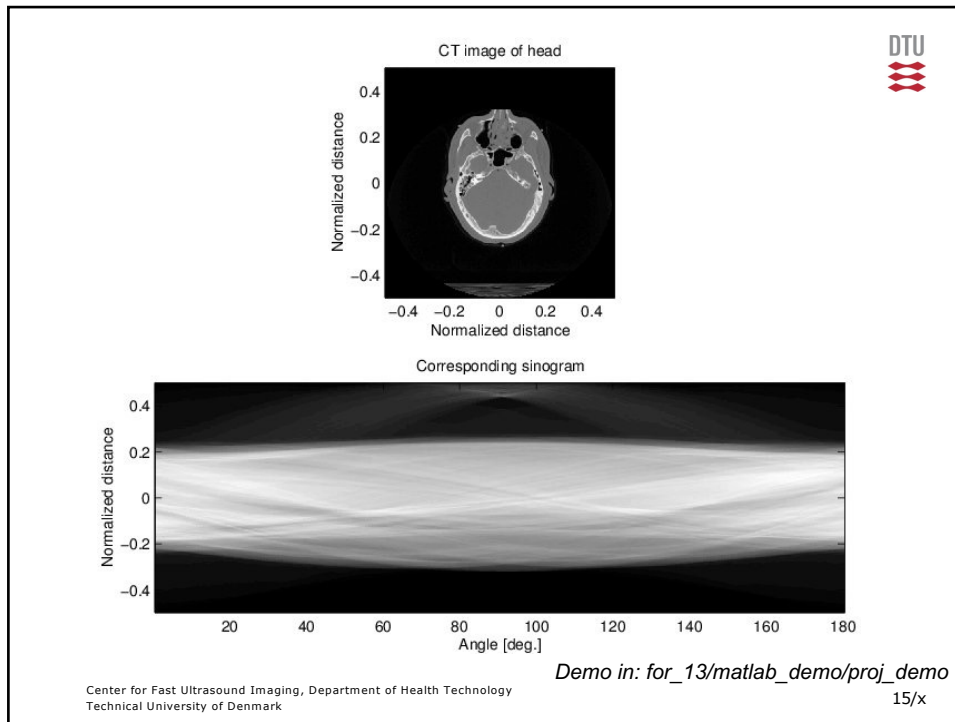
12



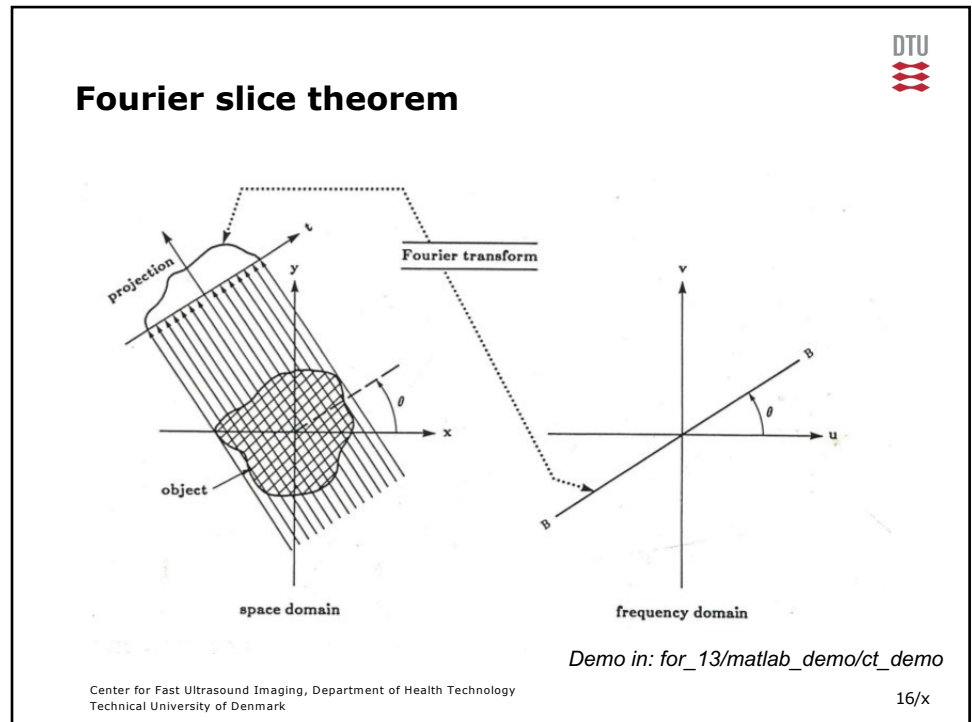
13



14



15



16



## Fourier Slice Theorem

$$P(\rho, \phi) = \int_{-\infty}^{+\infty} p(x', \phi) e^{-j2\pi\rho x'} dx'$$

$$= F(\rho \cos \phi, \rho \sin \phi)$$

$F(u, v)$  – 2D Fourier transform of image  
 $\phi$  – Gantry rotation

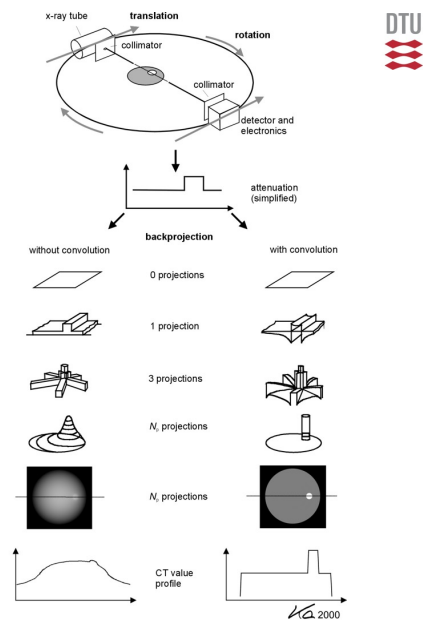
## Filtered Back Projection (FBP)

$$\hat{f}(x, y) = \int_0^{\pi} \int_{-\infty}^{+\infty} |\rho| P(\rho, \phi) e^{j2\pi\rho x'} d\rho d\phi$$

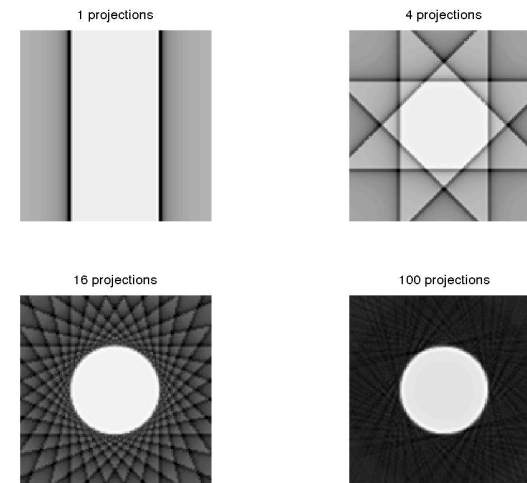
$\hat{f}(x, y)$  – Reconstructed image  
 $\phi$  – Gantry rotation  
 $x'$  – Detector position

## Filtered backprojection

- Perform for all projections:
  - Make Fourier transform of projected data
  - Apply filter in Fourier domain
  - Make inverse transform
  - Back-project and sum with previous image



## Influence from number of projections



## Ram-Lak filter

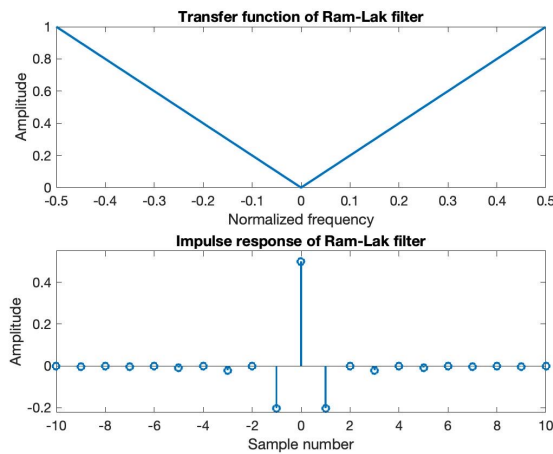


Transfer function:

$$h(\rho) = \begin{cases} |\rho|, & |\rho| \leq B \\ 0 & \text{else} \end{cases}$$

Impulse response

$$h(k) = \begin{cases} B^2 & k = 0 \\ -\frac{B^2}{\left(\frac{\pi}{2}k\right)^2} & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$$



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## Impulse response for Ram-Lak filter



- Impulse response of square wave:

$$h_{square}(x) = A2B \frac{\sin 2\pi B x}{2\pi B x}$$

- Ram-Lak impulse response: square wave – triangle:

$$h_{ramlak}(x) = 2BB \frac{\sin 2\pi B x}{2\pi B x} - B^2 \frac{\sin^2 \pi B x}{(\pi B x)^2}$$

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22/x

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## Sampling of impulse response for Ram-Lak filter



- Sampling at  $\frac{1}{\Delta x} = 2B, x = k \Delta x = \frac{k}{2B}$ :

$$h_{ramlak}(k) = 2B^2 \frac{\sin 2\pi B \frac{k}{2B}}{2\pi B \frac{k}{2B}} - B^2 \frac{\sin^2 \pi B \frac{k}{2B}}{(\pi B \frac{k}{2B})^2}$$

$$\sin 2\pi B \frac{k}{2B} = \sin \pi k = 0$$

$$\sin^2 \pi B \frac{k}{2B} = \sin^2 \frac{\pi}{2} k = 1 \text{ (k odd) else } 0$$

- Gives:

$$h(0) = B^2$$

$$h(k) = -B^2 \frac{\sin^2 k \frac{\pi}{2}}{(k \frac{\pi}{2})^2} = -\frac{B^2}{(k \frac{\pi}{2})^2} \text{ (k odd)}$$

$$h(k) = 0 \text{ else}$$

## Ram-Lak filter

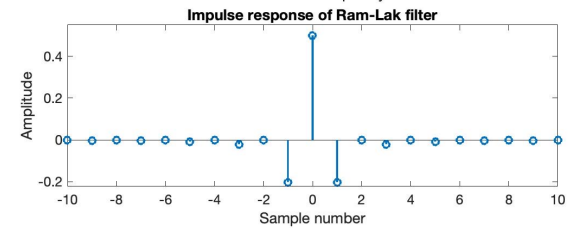
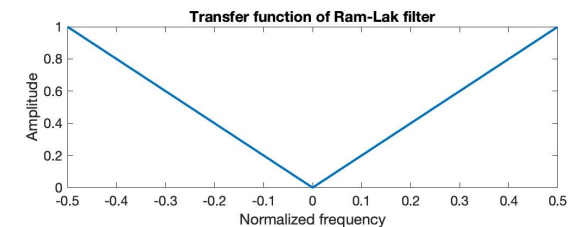


Transfer function:

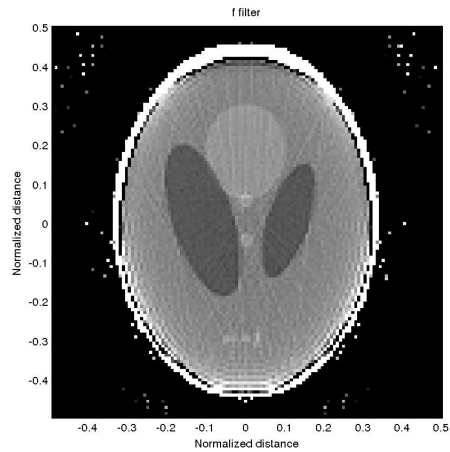
$$h(\rho) = \begin{cases} |\rho|, & |\rho| \leq B \\ 0 & \text{else} \end{cases}$$

Impulse response

$$h(k) = \begin{cases} B^2 & k = 0 \\ -\frac{B^2}{\left(\frac{\pi}{2}k\right)^2} & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$$



## Transfer function of filters – Ram-Lak



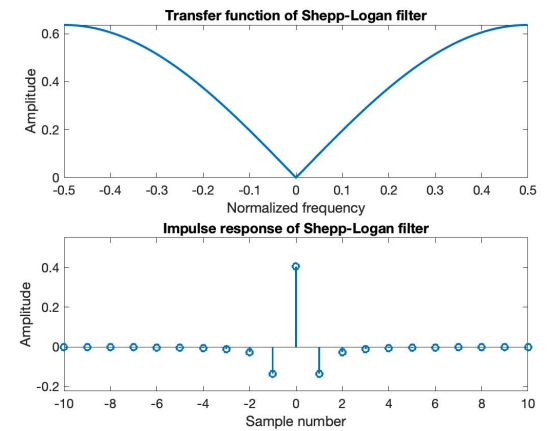
## Shepp-Logan filter

Transfer function:

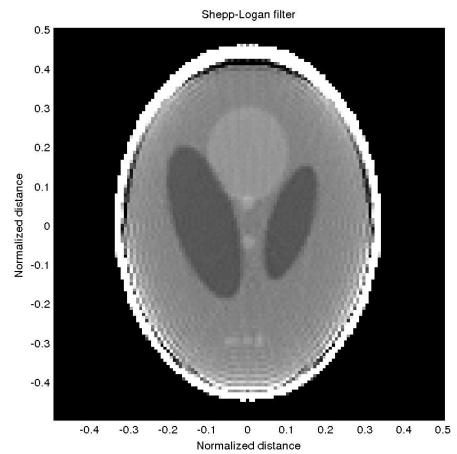
$$h(\rho) = \begin{cases} |\rho| \frac{\sin \frac{2\pi\rho}{4B}}{\frac{2\pi\rho}{4B}}, & |\rho| \leq B \\ 0 & \text{else} \end{cases}$$

Impulse response

$$h(k) = -\frac{8B^2}{\pi(4k^2 - 1)}$$



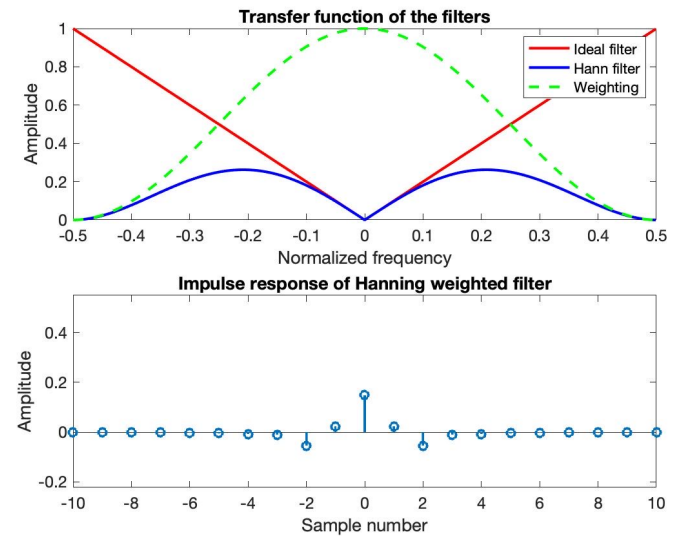
## Shepp-Logan filter



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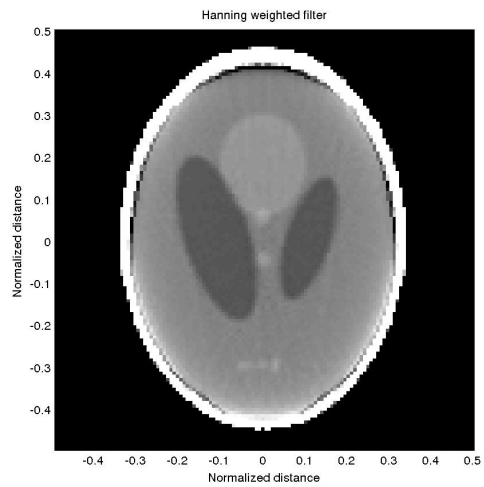
## Hanning weighted filter



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## Hanning weighted filter

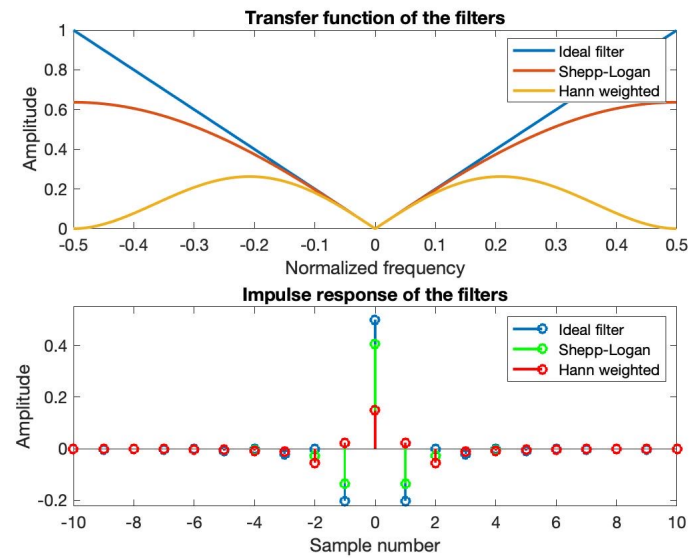


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29/x

29

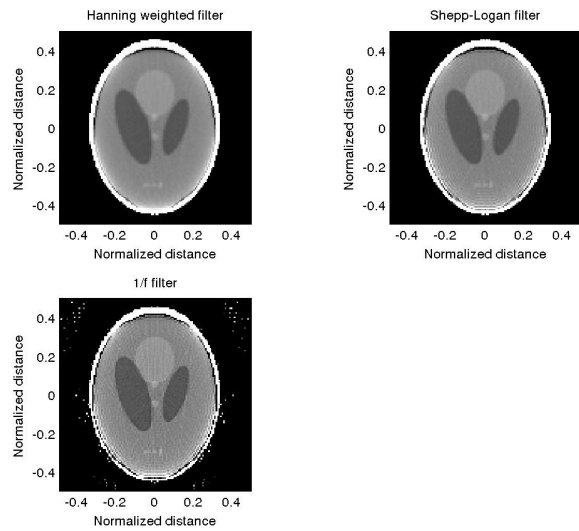
## Filter transfer functions and impulse responses



30/x

30

## Comparison between filters

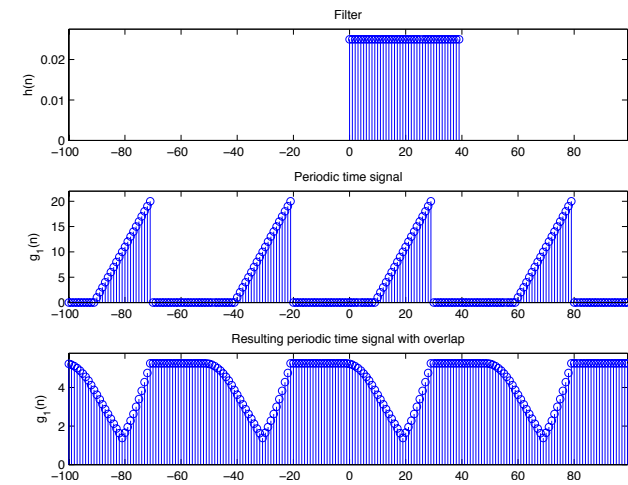


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31

## Circular convolution



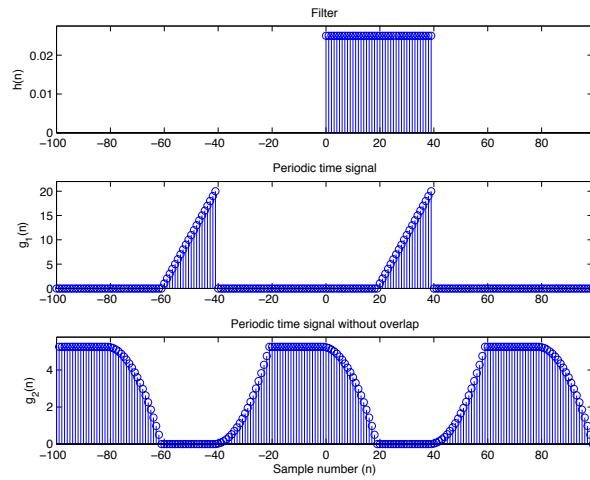
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32



## Circular convolution

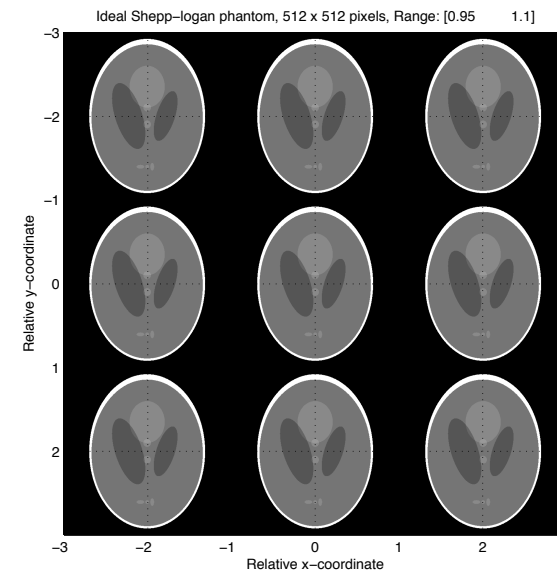


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33

## Circular convolution – Shepp-Logan



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# Backprojection

DTU

Patient grid

Projection

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35/x

# Data for testing and validation

- Use data sets on web site
- Circular phantom for geometry test
- Shepp-Logan for orientation and quantitative data
- In-vivo images for Hounsfield units

DTU

22485 Medical Imaging Systems

DTU Teaching Center for Fast Ultrasound Imaging Field II JAJ

CT data

On this page you can gain access to a database of CT images and projection data for the images. that can be used for testing reconstruction algorithms. Data for some phantom objects and various programs for artificial phantoms and data projections are also given here.

The clinical images shown on these pages have been released from the Visible Human Project. They were created from scanning a human cadaver with CT and MRI scanners and from subsequently aligning the cadaver into 1 mm sections for taking photographs.

A further description of the data can be found at here and a description of the program can be found here.

You can get to the different data pages by clicking on one of the images of the text below them.

Circle phantom

Shepp-Logan phantom

Image of head

Image of torso

Image of thorax

Data for assignments

A slide showing Hounsfield units for a CT scan can be downloaded here. The slide is taken from: E. Knefel (ed.): Imaging systems for medical diagnosis, Siemens, 1980.

A table of Mass Attenuation Coefficients and Mass Energy-Absorption Coefficients can be found at the following reference:  
<http://www.nsl.gov/pubs/nsl/ctdata/ctdata.htm>

A radiologic image atlas can be found at:  
<http://rad.usfca.miami.edu/homepage.html>

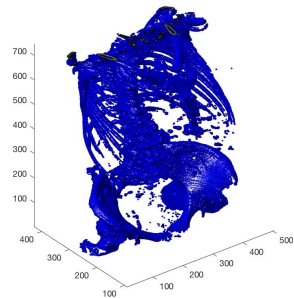
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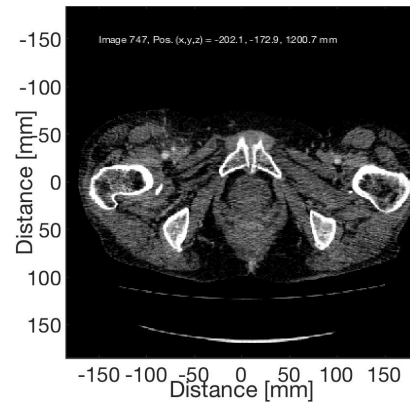
36/x

## Assignment data

- DICOM data from female patient
- All data available on the web
- Task is to find which slice it is



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Data and program in:  
[undervisning/k\\_22485\\_31545\\_billedder/ct\\_data/dicom\\_data](#)

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## Reading DICOM data

```
% Set overall parameters
dir_name='DICOM/ST00001/SE00001/'; % Directory name
start_image=1; % First image in series
end_image=747; % Last image in series
frame_rate=50; % Frame rate for playing back the movie

% Set the dynamic range for the display
off_set=100; % Offset [Hu]
range=400; % Range to display [Hu]
map_values=128; % Number of gray level values
bone_off_set= -250; % Offset for showing the bones
bone_range=100; % Range for showing the bones

% Initialize figure
colormap(gray(map_values));
dicom_movie(end_image+1-start_image) = struct('cdata',[],'colormap',[]);

% Read information for the first images
file_name='IM00001';
info=dicominfo([dir_name, file_name]);
dx=info.PixelSpacing(1);
dy=info.PixelSpacing(2);
Y = dicomread(info);
[Nx,Ny]=size(Y);

% Make space for all the images
Y=zeros(Nx,Ny,end_image+1-start_image);
z_positions=zeros(end_image+1-start_image,1);
```

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38



```
% Loop through the images and read and display them

for i=start_image:end_image
    file_name=['IM00',num2str(floor(i/100)),num2str(floor(rem(i,100)/10)),num2str(rem(i,10))];
    info=dicominfo([dir_name, file_name]);
    Y(:, :, i) = dicomread(info);
    image((1:Nx)-Nx/2)*dx, ((1:Ny)-Ny/2)*dy, (double(Y(:, :, i))+off_set)/range*map_values)
    xlabel('Distance [mm]')
    ylabel('Distance [mm]')
    pos=sprintf('%5.1f, %5.1f, %5.1f', info.ImagePositionPatient(1), ...
        info.ImagePositionPatient(2), info.ImagePositionPatient(3));
    z_positions(i)= info.ImagePositionPatient(3);
    text(-150, -150, ['Image ', num2str(i), ', Pos. (x,y,z) = ', pos, ' mm'], 'Color', [1 1 1])
    axis('image')
    drawnow
    dicom_movie(i)=getframe;
end

% Display the movie

movie(dicom_movie, 5, frame_rate);
```

Full script can be found at:

[courses.healthtech.dtu.dk/22485/files/ct\\_data/dicom\\_data/display\\_dicom\\_images.m](https://courses.healthtech.dtu.dk/22485/files/ct_data/dicom_data/display_dicom_images.m)

on the page for the CT data: [courses.healthtech.dtu.dk/22485/?ct\\_data/assign\\_data.html](https://courses.healthtech.dtu.dk/22485/?ct_data/assign_data.html)