Radon Transform and Filtered Backprojection

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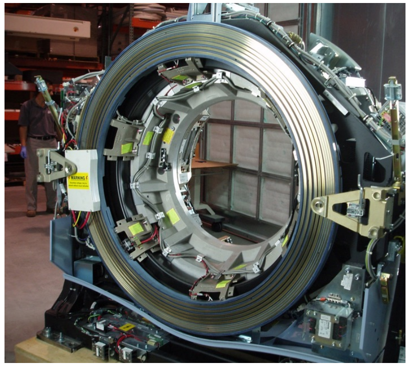
CT reconstruction - outline

- CT scanners
- Projection and Radon transform
- Projection demo

- Fourier slice theorem
- Inverse Radon transform – filtered backprojection
- Selection of filters
- Filtered backprojection algorithm

- Reading material: Prince & Links chapter 6



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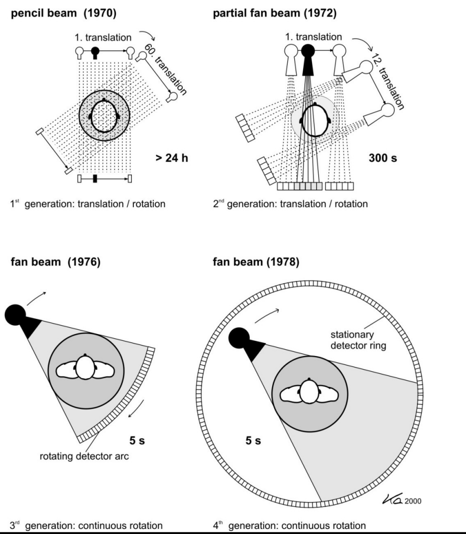
Modern CT system generations



From: W. A. Kalender, Computed Tomography, Publicis, 2005

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What do we measure?

- Intensity measured by detector:

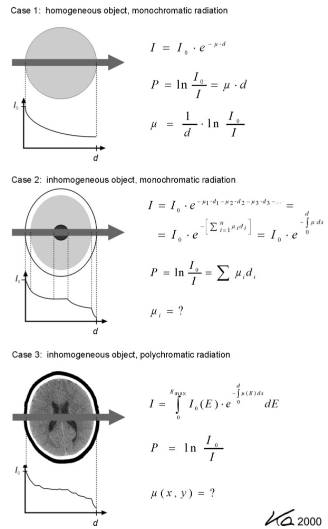
$$I = I_o \exp(-\mu \cdot x)$$

- Conversion to attenuation:

$$\mu = -\frac{1}{x}\ln\frac{I}{I_0}$$

- Attenuation values μ are scaled relative to water:

$$HU = \frac{\mu_{tissue} - \mu_{water}}{\mu_{water}} \times 1000$$



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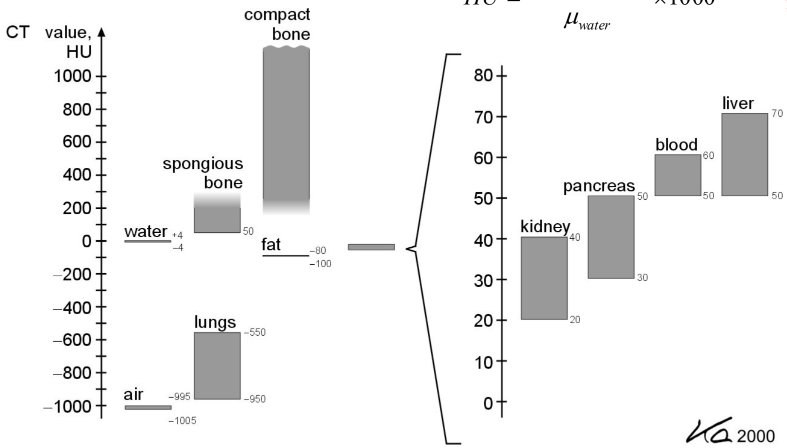
Hounsfield units

$$HU = \frac{\mu_{tissue} - \mu_{water}}{\mu_{water}} \times 1000$$



From: W. A. Kalender; Computed Tomography, Publicis, 2005

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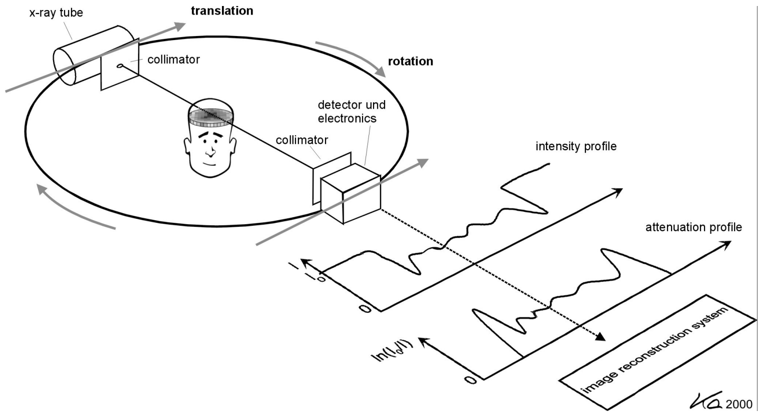
Measurement of attenuation



From: W. A. Kalender, Computer Tomography, Publicis, 2005

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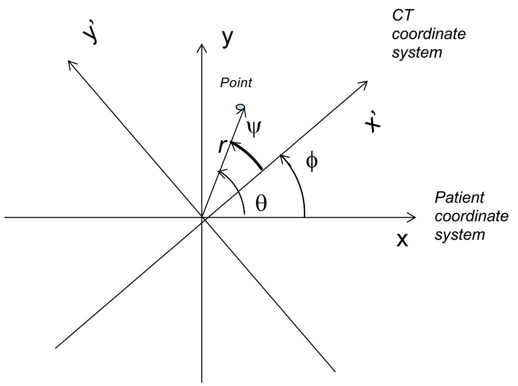
Parallel beam projection geometry



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Radon transform

$$p(x',\phi) = \int_{-\infty}^{+\infty} f(x'\cos\phi - y'\sin\phi, x'\sin\phi + y'\cos\phi)\,dy'$$

f(x,y) – Attenuation image
x', y' – Gantry coordinate system
x, y – Patient coordinate system

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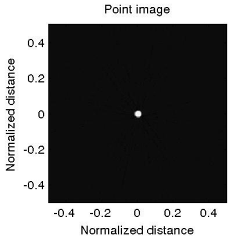
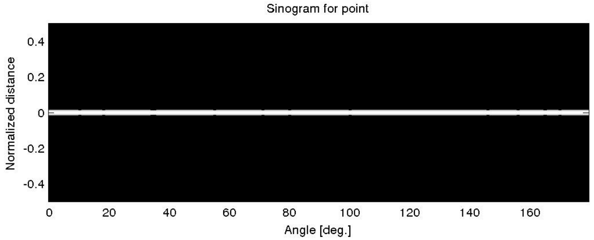
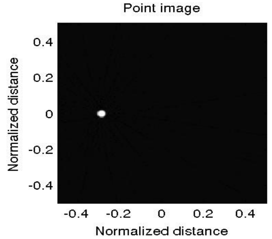
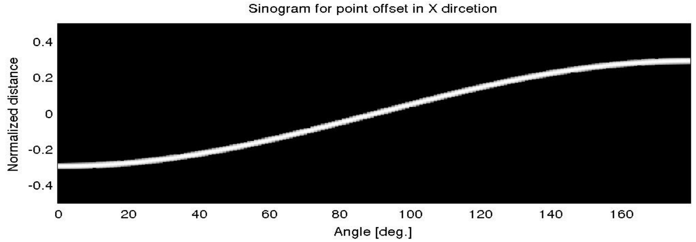
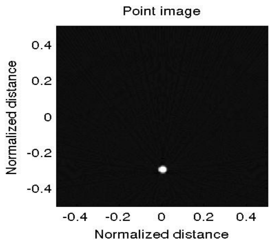
Sinogram for point

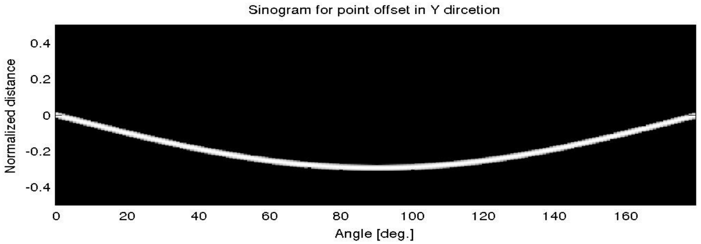


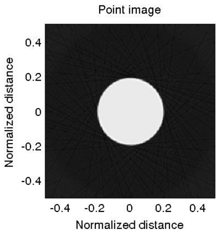


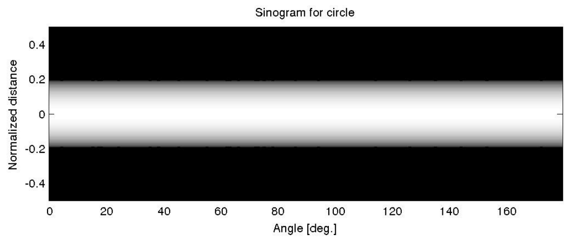


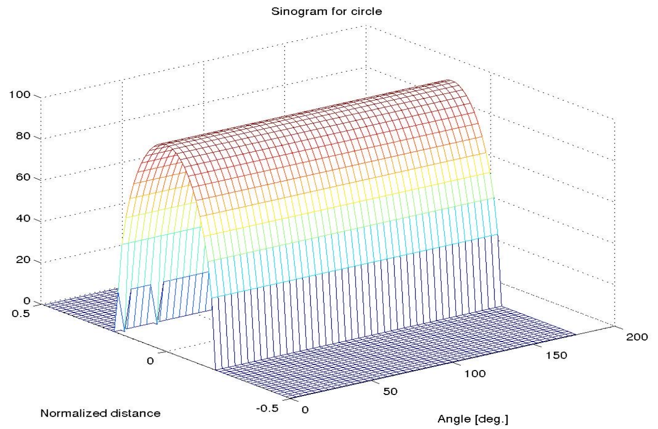












Sinogram for circle
100
80
60
40
20
0
0.5
0
-0.5
Normalized distance
0
50
100
150
200
Angle [deg.]

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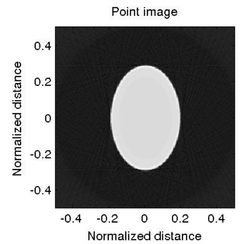
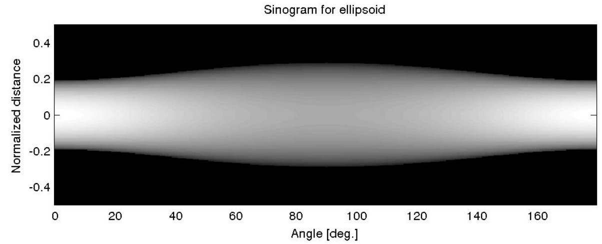


Point image
0.4
0.2
0
-0.2
-0.4
Normalized distance
-0.4 -0.2 0 0.2 0.4
Normalized distance



Sinogram for ellipsoid
0.4
0.2
0
-0.2
-0.4
Normalized distance
0 20 40 60 80 100 120 140 160
Angle [deg.]

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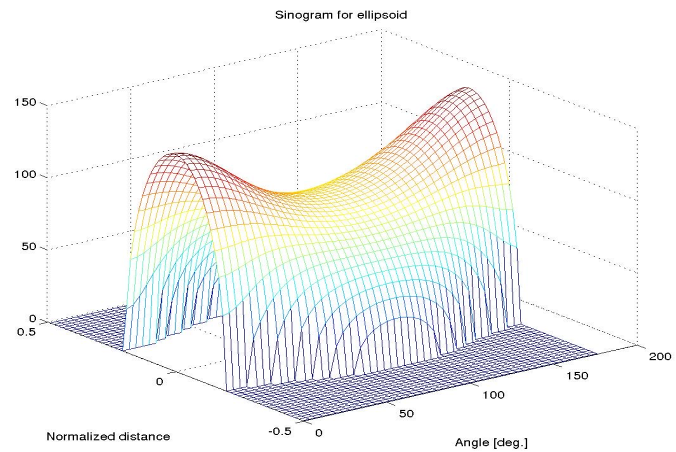

Sinogram for ellipsoid
150
100
50
0
0.5
0
-0.5
Normalized distance
0
50
100
150
200
Angle [deg.]

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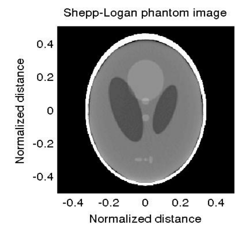
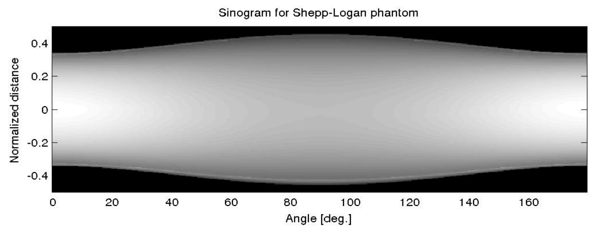


Shepp-Logan phantom image
0.4
0.2
0
-0.2
-0.4
Normalized distance
-0.4 -0.2 0 0.2 0.4
Normalized distance



Sinogram for Shepp-Logan phantom
0.4
0.2
0
-0.2
-0.4
Normalized distance
0 20 40 60 80 100 120 140 160
Angle [deg.]

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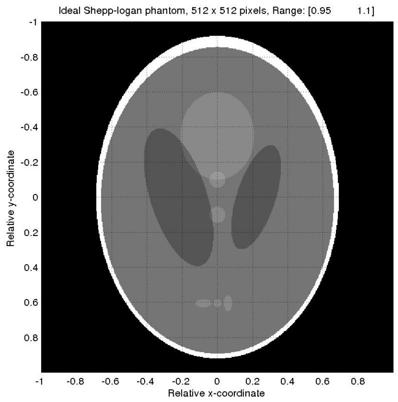
Shepp-Logan phantom



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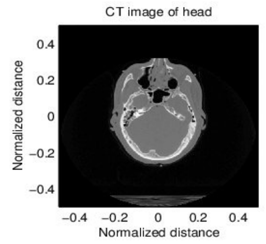

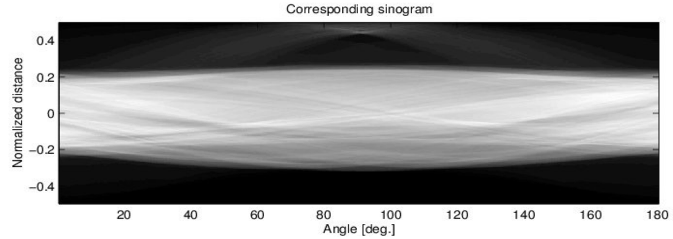

Demo in: for_13/matlab_demo/proj_demo

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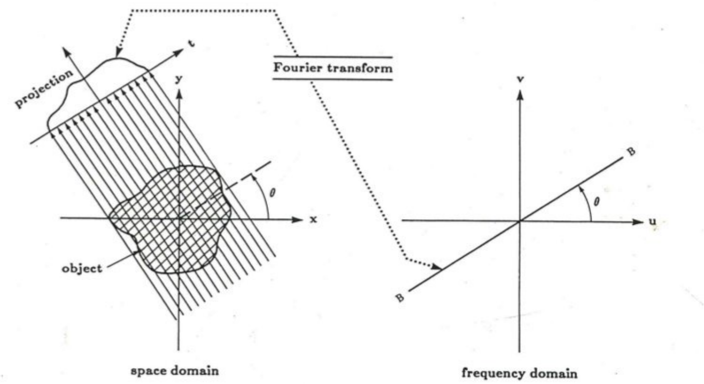
Fourier slice theorem



Demo in: for_13/matlab_demo/ct_demo

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Fourier Slice Theorem

$$P(\rho,\phi) = \int_{-\infty}^{+\infty} p(x',\phi) e^{-j2\pi\rho x'} dx'$$

$$= F(\rho\cos\phi, \rho\sin\phi)$$

F(u,v) – 2D Fourier transform of image

ϕ – Gantry rotation

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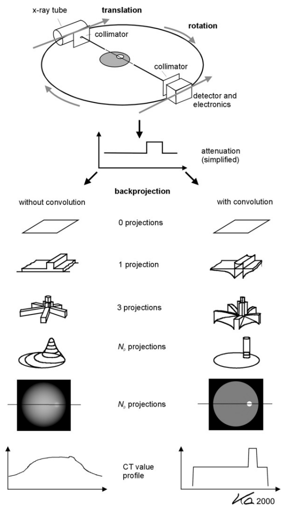
Filtered Back Projection (FBP)

$$\hat{f}(x,y) = \int_{0}^{\pi} \int_{-\infty}^{+\infty} |\rho| P(\rho,\phi) e^{j2\pi\rho x'} d\rho d\phi$$

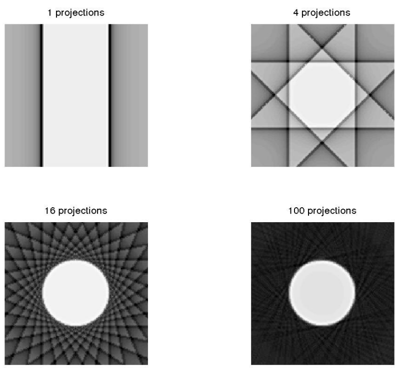
$\hat{f}(x,y)$ – Reconstructed image
ϕ – Gantry rotation
x' – Detector position

Filtered backprojection

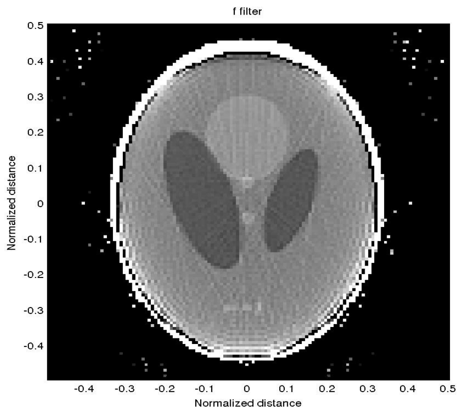
- Perform for all projections:
  1. Make Fourier transform of projected data
  2. Apply filter in Fourier domain
  3. Make invers Fourier transform
  4. Backproject and sum with previous image



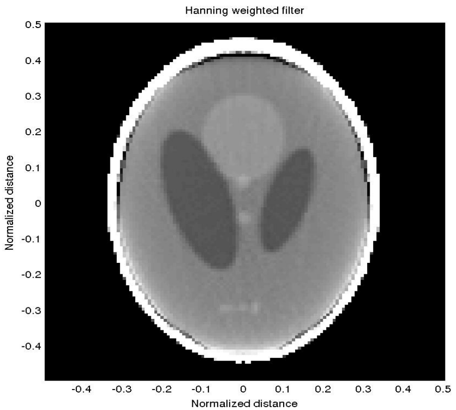
Influence from number of projections



Transfer function of filters - Ideal



Hanning weighted filter



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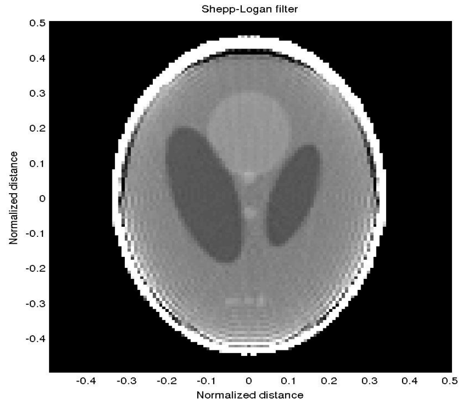
Shepp-Logan filter



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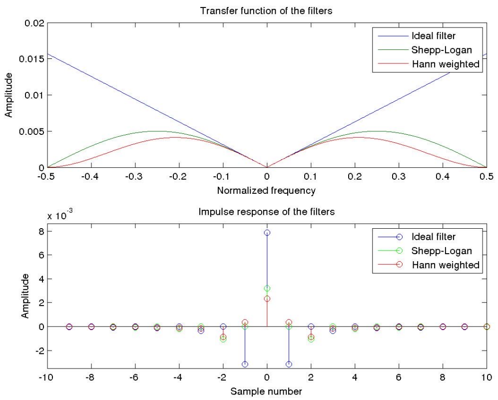
Filter transfer functions and impulse responses



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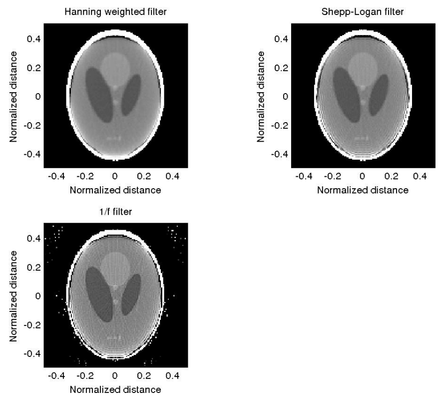
Comparison between filters



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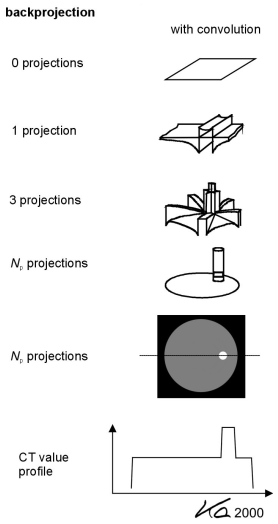
Filtered backprojection

- Perform for all projection:
  1. Make Fourier transform of projected data
  2. Apply filter in Fourier domain
  3. Make invers transform
  4. Backproject and sum with previous image



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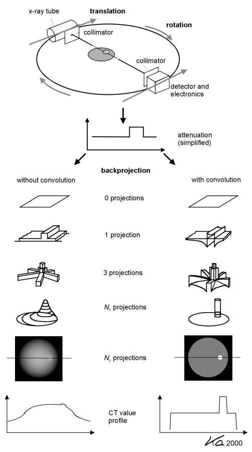
Summary

- Parallel beam projection and Radon transform
- Fourier slice theorem
- Filtered backprojection reconstruction and choices
- P & L: Chapter 6
- Questions for CT assignments



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Reconstruction in CT: Hints for the assignments

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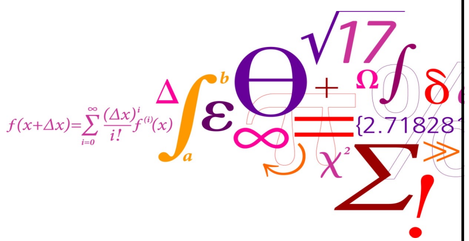

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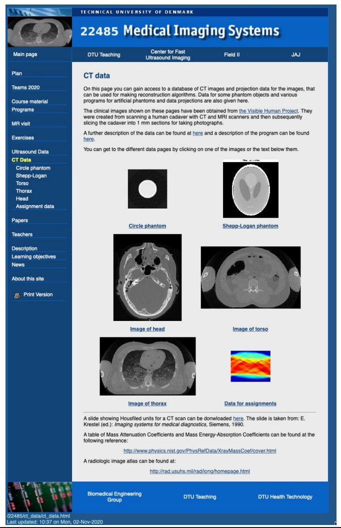
Data for testing and validation

- Use data sets on web site
- Circular phantom for geometry test
- Shepp-Logan for orientation and quantitative data
- In-vivo images for Hounsfield units



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32/x

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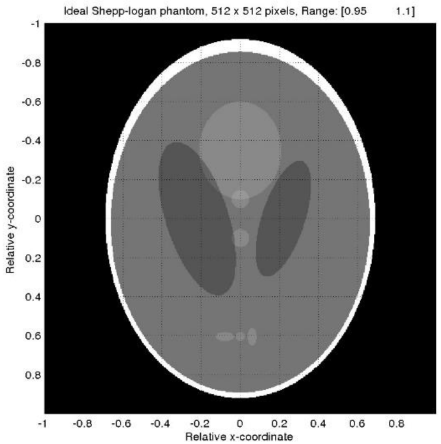
Shepp-Logan phantom



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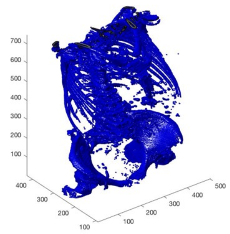
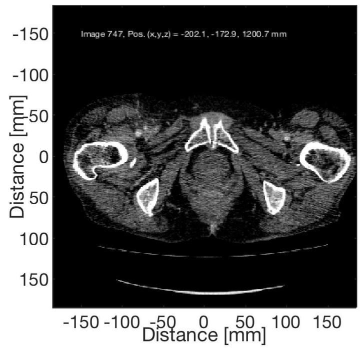
Assignment data

- DICOM data from female patient
- All data available on the web
- Task is to find which slice it is





https://courses.healthtech.dtu.dk/22485/?ct_data/assign_data.html

Data and program in: undervisning/k_22485_31545_billeder/ct_data/dicom_data

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Reading DICOM data 1

```
%  Set overall parameters
dir_name='DICOM/ST00001/SE00001/';  %  Directory name
start_image=1;                      %  First image in series
end_image=747;                      %  Last image in series
frame_rate=50;                      %  Frame rate for playing back the movie

%  Set the dynamic range for the display
off_set=100;         %  Offset [Hu]
range=400;           %  Range to display [Hu]
map_values=128;      %  Number of gray level values
bone_off_set= -250;  %  Offset for showing the bones
bone_range=100;      %  Range for showing the bones

%  Initialize figure
colormap(gray(map_values));
dicom_movie(end_image+1-start_image) = struct('cdata',[],'colormap',[]);
```


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Reading DICOM data 2

```
Read information for the first images
file_name='IM00001';
info=dicominfo([dir_name, file_name]);
dx=info.PixelSpacing(1);
dy=info.PixelSpacing(2);
Y = dicomread(info);
[Nx,Ny]=size(Y);

%  Make space for all the images
Y=zeros(Nx,Ny,end_image+1-start_image);
z_positions=zeros(end_image+1-start_image,1);
```


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Reading DICOM data 3

```
%  Loop through the images and read and display them
for i=start_image:end_image
    file_name=['IM00',num2str(floor(i/100)),num2str(floor(rem(i,100)/10)),num2str(rem(i,10))];
    info=dicominfo([dir_name, file_name]);
    Y(:,:,i) = dicomread(info);
    image(((1:Nx)-Nx/2)*dx,((1:Ny)-Ny/2)*dy,(double(Y(:,:,i))+off_set)/range*map_values)
    xlabel('Distance [mm]')
    ylabel('Distance [mm]')
    pos=sprintf('%5.1f, %5.1f, %5.1f', info.ImagePositionPatient(1), …
         info.ImagePositionPatient(2), info.ImagePositionPatient(3));
    z_positions(i)= info.ImagePositionPatient(3);
    text(-150, -150, ['Image ', num2str(i),', Pos. (x,y,z) = ', pos,' mm'], 'Color', [1 1 1])
    axis('image')
    drawnow
    dicom_movie(i)=getframe;
end
%  Display the movie
movie(dicom_movie, 5, frame_rate);
```


Script in: ct_data/dicom_data

Full script can be found at:

https://courses.healthtech.dtu.dk/22485/files/ct_data/dicom_data/display_dicom_images.m

on the page for the CT data: https://courses.healthtech.dtu.dk/22485/?ct_data/assign_data.html