Radon Transform and Filtered Backprojection

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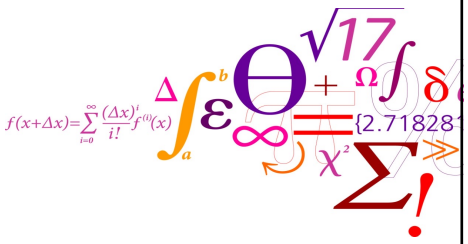

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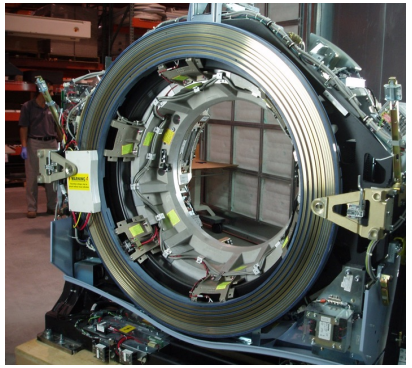
1

CT reconstruction - outline

- CT scanners
- Projection and Radon transform
- Projection demo

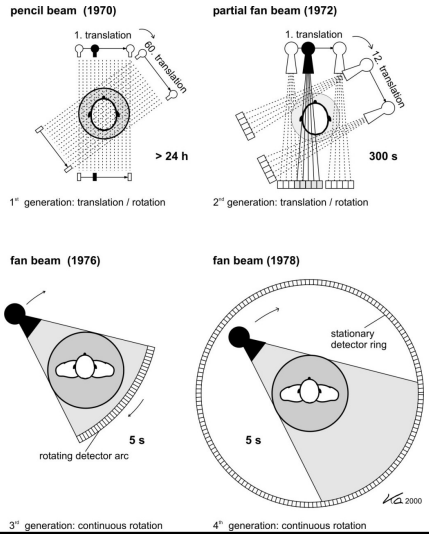
- Fourier slice theorem
- Inverse Radon transform – filtered backprojection
- Selection of filters
- Filtered backprojection algorithm

- Reading material: Prince & Links chapter 6



2

Modern CT system generations



From: W. A. Kalender: Computed Tomography, Publicis, 2005

What do we measure?

- Intensity measured by detector:

$$I = I_o \exp(-\mu \cdot x)$$

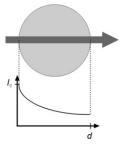
- Conversion to attenuation:

$$\mu = -\frac{1}{x} \ln \frac{I}{I_0}$$

- Attenuation values μ are scaled relative to water:

$$HU = \frac{\mu_{tissue} - \mu_{water}}{\mu_{water}} \times 1000$$

Case 1: homogeneous object, monochromatic radiation

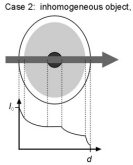


$$I = I_0 \cdot e^{-\mu \cdot d}$$

$$P = \ln \frac{I_0}{I} = \mu \cdot d$$

$$\mu = \frac{1}{d} \cdot \ln \frac{I_0}{I}$$

Case 2: inhomogeneous object, monochromatic radiation

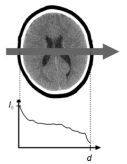


$$I = I_0 \cdot e^{-\mu_1 \cdot d_1 - \mu_2 \cdot d_2 - \mu_3 \cdot d_3 - \ldots} = I_0 \cdot e^{-\left[\sum_{i=1}^{n} \mu_i d_i\right]} = I_0 \cdot e^{-\int_0^d \mu \, dx}$$

$$P = \ln \frac{I_0}{I} = \sum \mu_i d_i$$

$\mu_i = ?$

Case 3: inhomogeneous object, polychromatic radiation



$$I = \int_0^{E_{max}} I_0(E) \cdot e^{-\int_0^d \mu(E) dx} dE$$

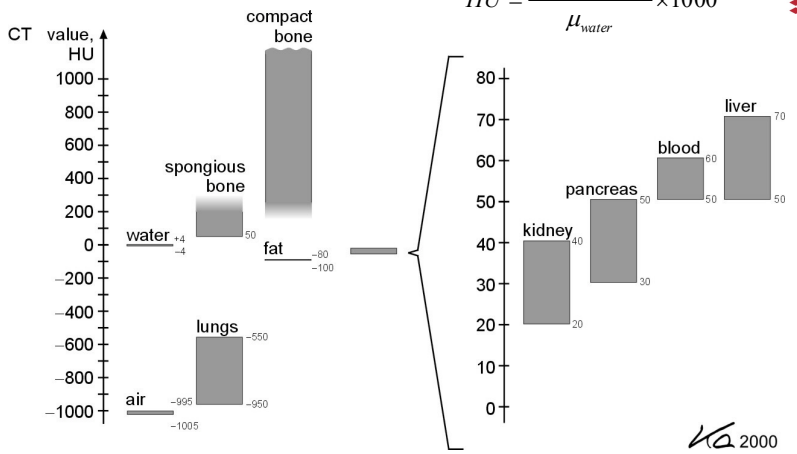
$$P = \ln \frac{I_0}{I}$$

$\mu(x, y) = ?$

2000

Hounsfield units

$$HU = \frac{\mu_{tissue} - \mu_{water}}{\mu_{water}} \times 1000$$



From: W. A. Kalender; Computed Tomography, Publicis, 2005

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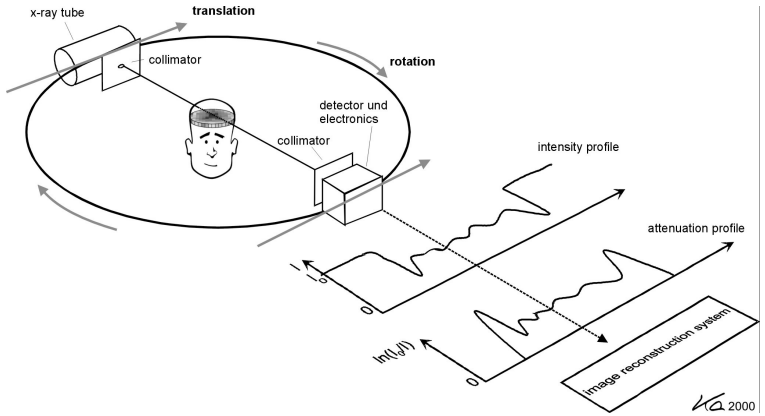
Measurement of attenuation



From: W. A. Kalender; Computer Tomography, Publicis, 2005

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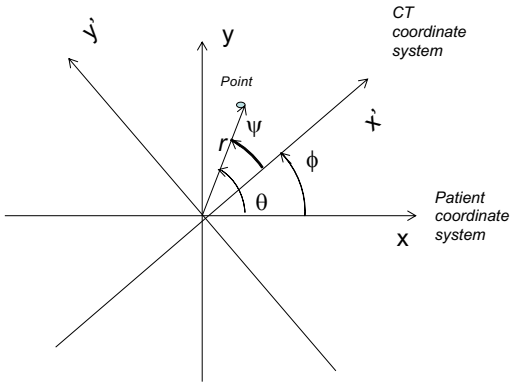
Parallel beam projection geometry



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Radon transform

$$p(x',\phi) = \int_{-\infty}^{+\infty} f(x'\cos\phi - y'\sin\phi, x'\sin\phi + y'\cos\phi)dy'$$

f(x,y) – Attenuation image
x', y' – Gantry coordinate system
x, y – Patient coordinate system

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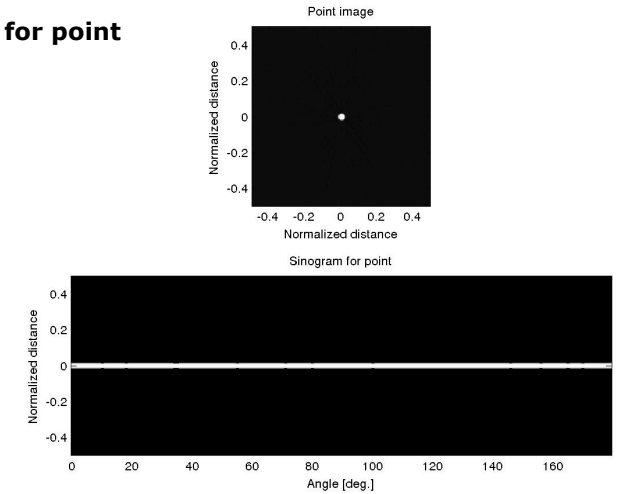
Sinogram for point



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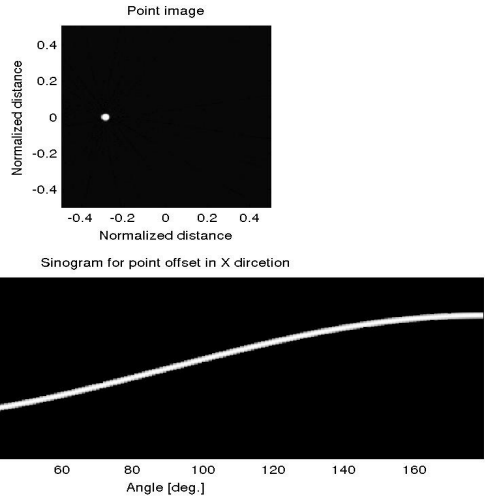

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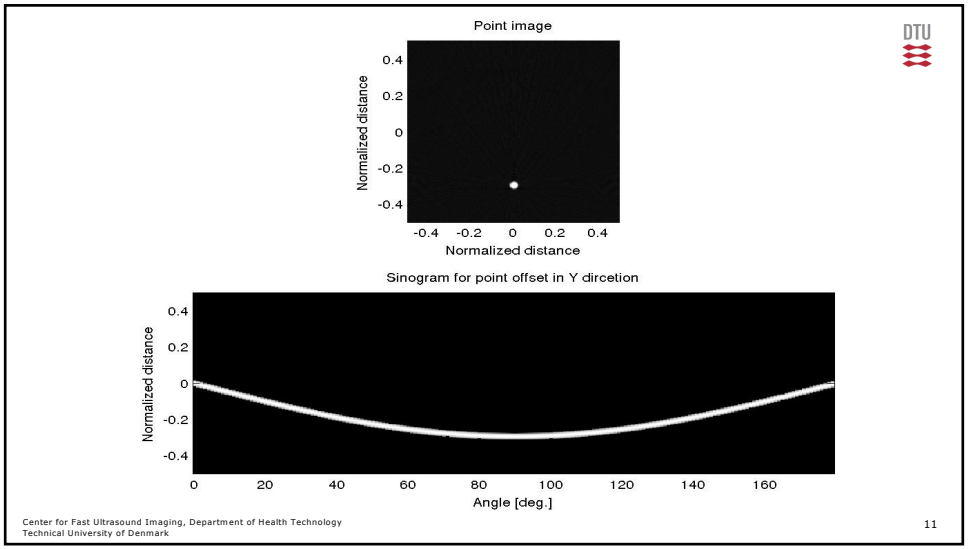
Point image
Normalized distance
0.4
0.2
0
-0.2
-0.4
-0.4 -0.2 0 0.2 0.4
Normalized distance
Sinogram for point offset in Y dircetion
Normalized distance
0.4
0.2
0
-0.2
-0.4
0 20 40 60 80 100 120 140 160
Angle [deg.]
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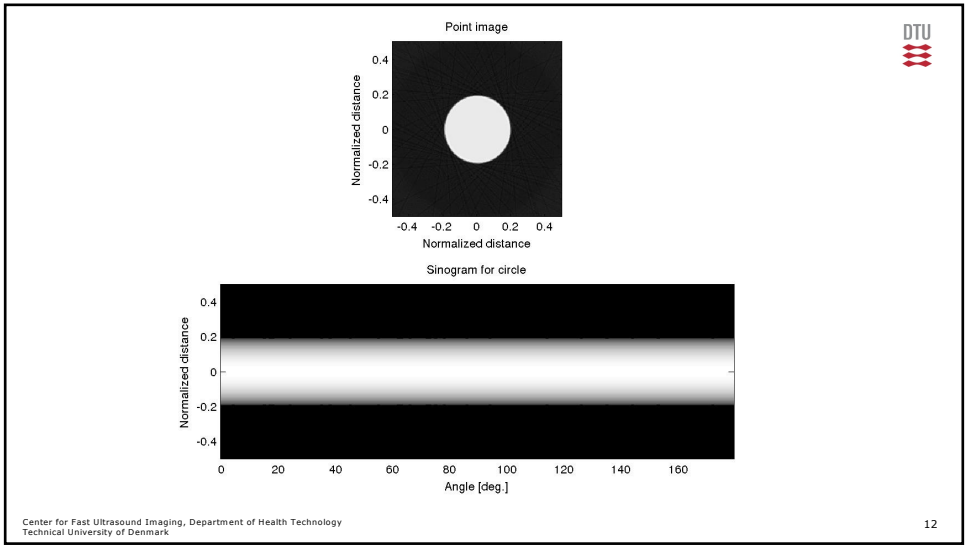
Point image
Normalized distance
0.4
0.2
0
-0.2
-0.4
-0.4 -0.2 0 0.2 0.4
Normalized distance
Sinogram for circle
Normalized distance
0.4
0.2
0
-0.2
-0.4
0 20 40 60 80 100 120 140 160
Angle [deg.]
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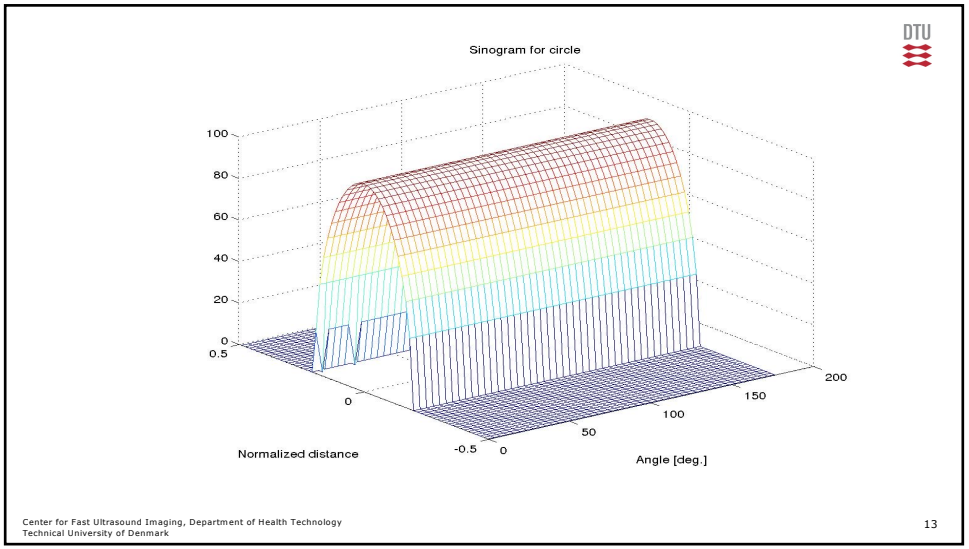
Sinogram for circle
100
80
60
40
20
0
0.5
0
-0.5
Normalized distance
0
50
100
150
200
Angle [deg.]
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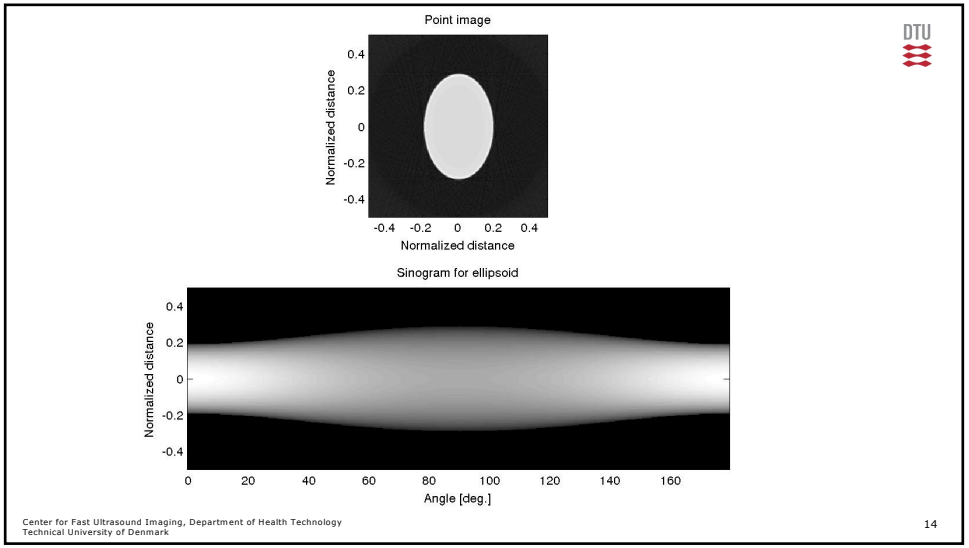
Point image
0.4
0.2
0
-0.2
-0.4
Normalized distance
-0.4 -0.2 0 0.2 0.4
Normalized distance
Sinogram for ellipsoid
0.4
0.2
0
-0.2
-0.4
Normalized distance
0 20 40 60 80 100 120 140 160
Angle [deg.]
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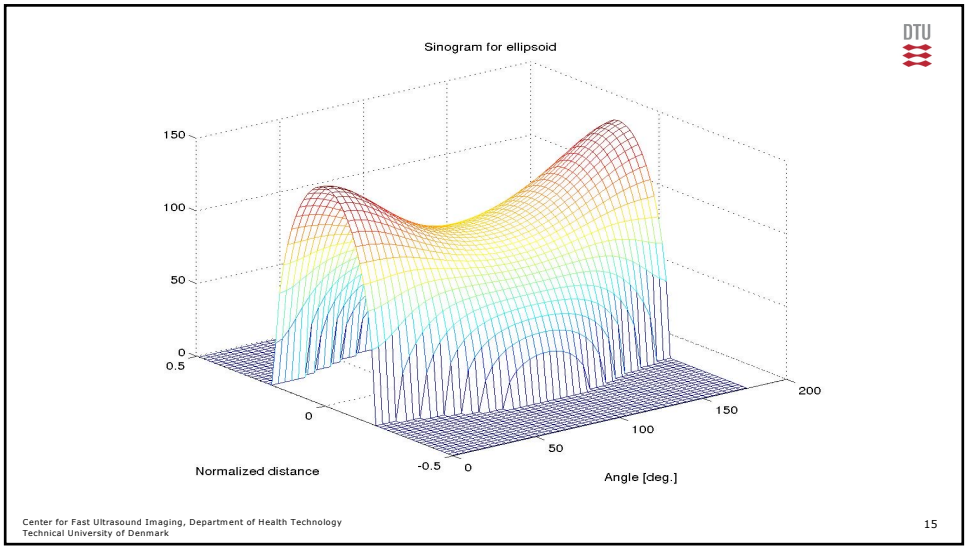
Sinogram for ellipsoid
150
100
50
0
0.5
0
-0.5
Normalized distance
0
50
100
150
200
Angle [deg.]
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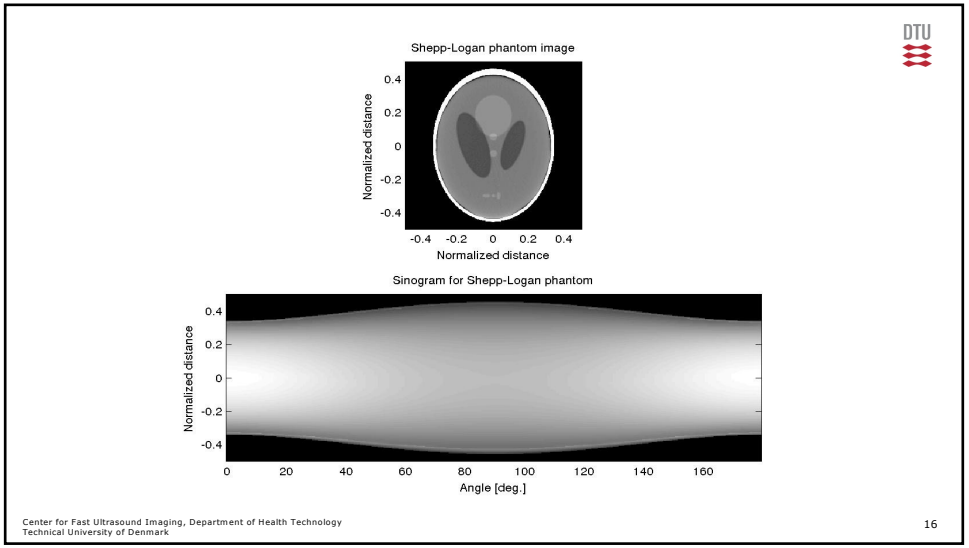
Shepp-Logan phantom image
0.4
0.2
0
-0.2
-0.4
Normalized distance
-0.4 -0.2 0 0.2 0.4
Normalized distance
Sinogram for Shepp-Logan phantom
0.4
0.2
0
-0.2
-0.4
Normalized distance
0 20 40 60 80 100 120 140 160
Angle [deg.]
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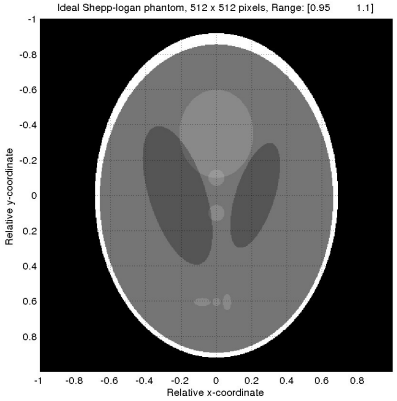
Shepp-Logan phantom



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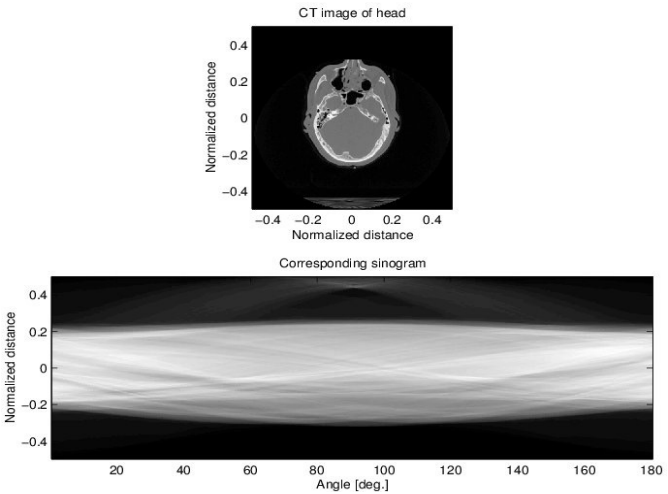

Demo in: for_13/matlab_demo/proj_demo

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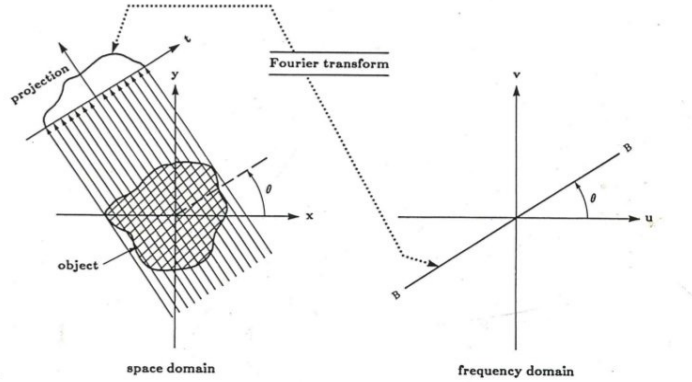
Fourier slice theorem



Demo in: for_13/matlab_demo/ct_demo

Fourier Slice Theorem

$$P(\rho,\phi) = \int_{-\infty}^{+\infty} p(x',\phi) e^{-j2\pi\rho x'}\, dx'$$

$$= F(\rho\cos\phi, \rho\sin\phi)$$

F(u,v) – 2D Fourier transform of image
ϕ – Gantry rotation

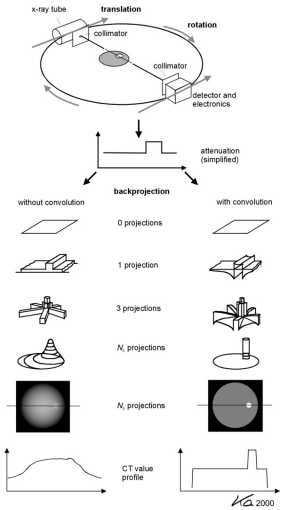
Filtered Back Projection (FBP)

$$\hat{f}(\mathrm{x},\mathrm{y}) = \int_{0}^{\pi} \int_{-\infty}^{+\infty} |\rho| P(\rho,\phi) e^{j2\pi\rho x'} d\rho d\phi$$

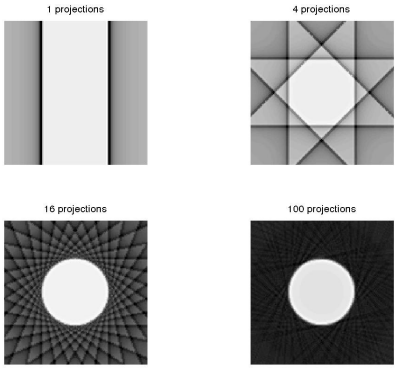
$\hat{f}(\mathrm{x},\mathrm{y})$ – Reconstructed image
ϕ – Gantry rotation
x' – Detector position

Filtered backprojection

- Perform for all projections:
  1. Make Fourier transform of projected data
  2. Apply filter in Fourier domain
  3. Make invers Fourier transform
  4. Backproject and sum with previous image



Influence from number of projections



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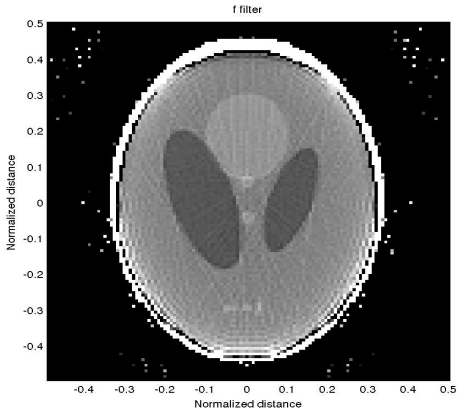
Transfer function of filters - Ideal



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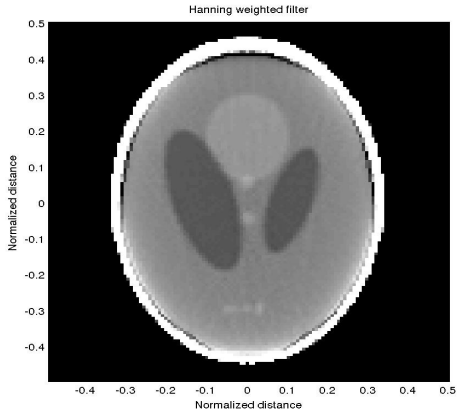
Hanning weighted filter



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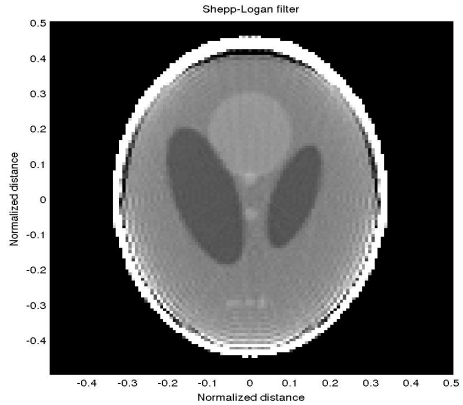
Shepp-Logan filter



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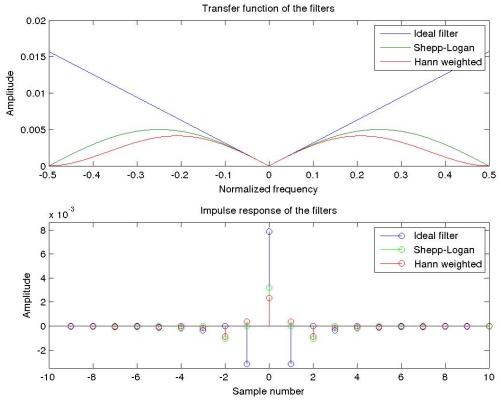
Filter transfer functions and impulse responses



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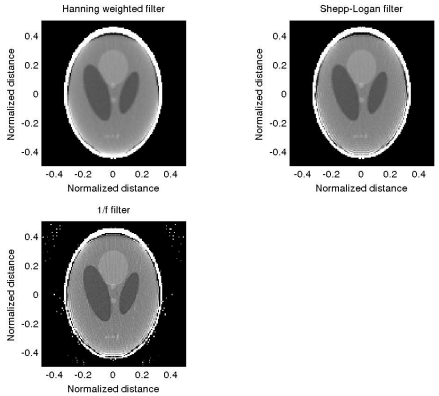
Comparison between filters



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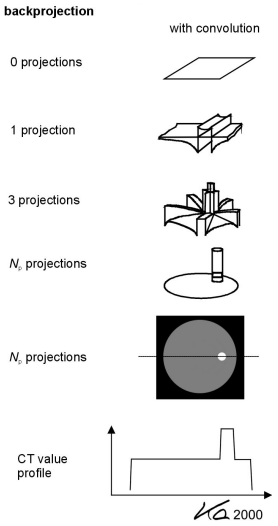
Filtered backprojection

- Perform for all projection:
  1. Make Fourier transform of projected data
  2. Apply filter in Fourier domain
  3. Make invers transform
  4. Backproject and sum with previous image



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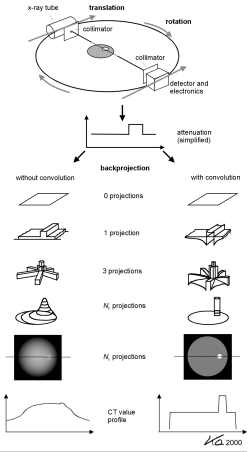
Summary

- Parallel beam projection and Radon transform
- Fourier slice theorem
- Filtered backprojection reconstruction and choices
- P & L: Chapter 6
- Questions for CT assignments



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Reconstruction in CT: Hints for the assignments

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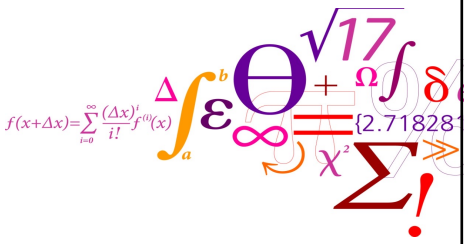

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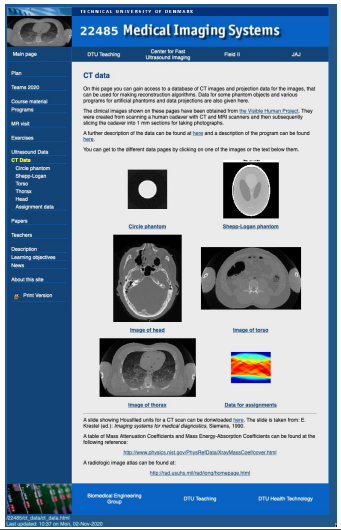
Data for testing and validation

- Use data sets on web site
- Circular phantom for geometry test
- Shepp-Logan for orientation and quantitative data
- In-vivo images for Hounsfield units



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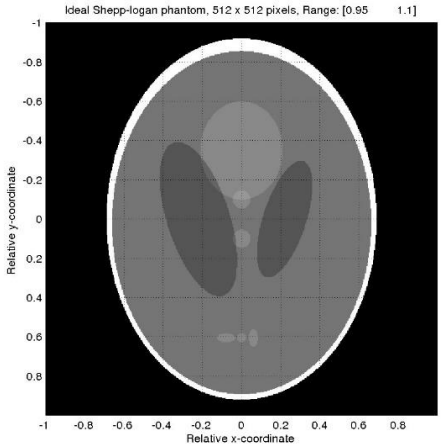
Shepp-Logan phantom



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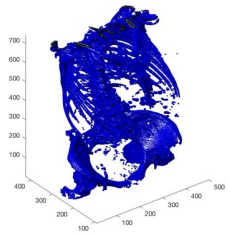
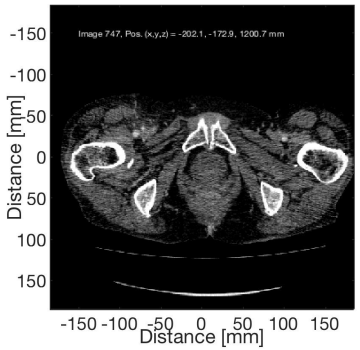
Assignment data

- DICOM data from female patient
- All data available on the web
- Task is to find which slice it is





https://courses.healthtech.dtu.dk/22485/?ct_data/assign_data.html

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Data and program in: undervisning/k_22485_31545_billeder/ct_data/dicom_data 34/x

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Reading DICOM data 1

```
%  Set overall parameters
dir_name='DICOM/ST00001/SE00001/';  %  Directory name
start_image=1;                      %  First image in series
end_image=747;                      %  Last image in series
frame_rate=50;                      %  Frame rate for playing back the movie

%  Set the dynamic range for the display
off_set=100;          %  Offset [Hu]
range=400;            %  Range to display [Hu]
map_values=128;       %  Number of gray level values
bone_off_set= -250;   %  Offset for showing the bones
bone_range=100;       %  Range for showing the bones

%  Initialize figure
colormap(gray(map_values));
dicom_movie(end_image+1-start_image) = struct('cdata',[],'colormap',[]);
```


Reading DICOM data 2

```
Read information for the first images
file_name='IM00001';
info=dicominfo([dir_name, file_name]);
dx=info.PixelSpacing(1);
dy=info.PixelSpacing(2);
Y = dicomread(info);
[Nx,Ny]=size(Y);

%  Make space for all the images
Y=zeros(Nx,Ny,end_image+1-start_image);
z_positions=zeros(end_image+1-start_image,1);
```

Reading DICOM data 3

```
%  Loop through the images and read and display them
for i=start_image:end_image
    file_name=['IM00',num2str(floor(i/100)),num2str(floor(rem(i,100)/10)),num2str(rem(i,10))];
    info=dicominfo([dir_name, file_name]);
    Y(:,:,i) = dicomread(info);
    image(((1:Nx)-Nx/2)*dx,((1:Ny)-Ny/2)*dy,(double(Y(:,:,i))+off_set)/range*map_values)
    xlabel('Distance [mm]')
    ylabel('Distance [mm]')
    pos=sprintf('%5.1f, %5.1f, %5.1f', info.ImagePositionPatient(1), …
         info.ImagePositionPatient(2), info.ImagePositionPatient(3));
    z_positions(i)= info.ImagePositionPatient(3);
    text(-150, -150, ['Image ', num2str(i),', Pos. (x,y,z) = ', pos,' mm'], 'Color', [1 1 1])
    axis('image')
    drawnow
    dicom_movie(i)=getframe;
end
%  Display the movie
movie(dicom_movie, 5, frame_rate);
```


Full script can be found at:

https://courses.healthtech.dtu.dk/22485/files/ct_data/dicom_data/display_dicom_images.m

on the page for the CT data: https://courses.healthtech.dtu.dk/22485/?ct_data/assign_data.html