

Assignment A2

“Masterwork” in pixels, voxels & spatial resolution size

Jens E. Wilhjelm

Biomedical Instrumentation, Department of Health Technology,
Ørstedes plads, building 349
Technical University of Denmark

2800 Kgs. Lyngby

(Ver. 3.4 24/8/21) © 2018-2021 by JW

1 Introduction

This document presents a number of open-ended problems that might require some help. Therefore, you are encouraged to solve them under supervision while your group is waiting for its turn to experience the medical scanners at Bispebjerg Hospital and also the following Thursday. Since learning is best achieved by being active yourself and as medical imaging often must be understood by visualisation, this self-tutorial will lead you through a number of drawing exercises.

Specifically, this guide on X-ray will walk you through the concepts of pixels (picture elements), voxels (volume elements), spatial resolution size as well as some considerations on how X-ray pictures can be interpreted.

2 2D projection

First consider a conventional X-ray system as shown in Figure 1. If you are new to X-ray, just think of the shadow caused by visible light (from a very small source) penetrating a partly transparent object. The light is made of photons (= elementary particles of light) that normally travels along straight lines.

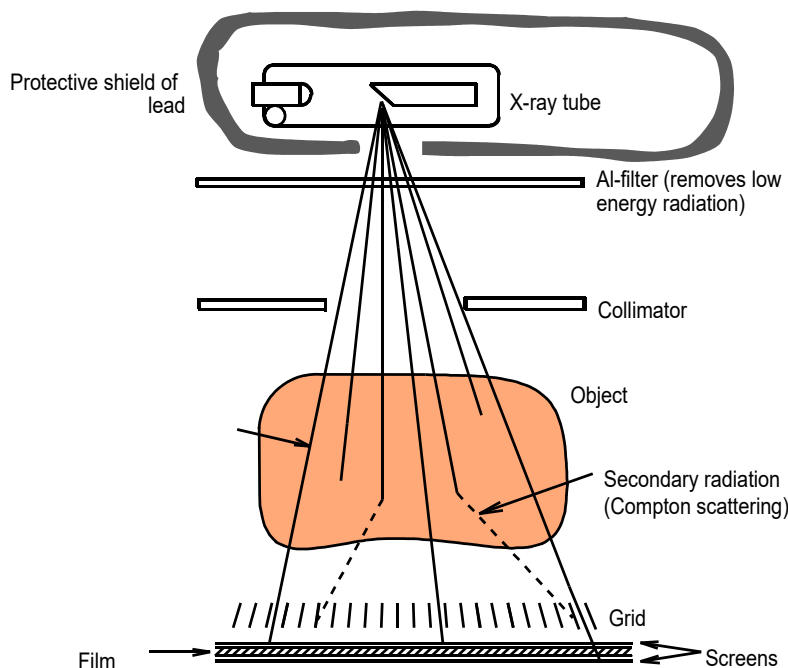


Figure 1 A typical planar X-ray system.

The X-ray system works this way:

- The X-rays are generated by the X-ray tube (Danish: *Røntgenrør*).
- The light first goes through an Al filter (aluminium plate) which removes low energy photons that are not able to penetrate the object and, thus, not able to contribute to the information on the film. Therefore, it just constitutes a needless dose of radiation to the object.
- X-ray radiation outside the image region on the film is removed by the collimator (in Danish: *primærblænde*).
- *Attenuation* and *Compton scattering* take place at the object. Attenuation is taking place all over the object (the higher the density, the larger the attenuation). Compton scattering change the *direction* of the photons. However, only photons moving directly in a straight line from the source to the film/detector are allowed through the grid at the bottom (in Danish: *sekundærblænde*). Thus, it is only the attenuation that is measured on the film.
- The last component of the system is the X-ray detector or “film”. In the childhood of X-ray systems, only a so-called “film” was available; this was sensitive to the intensity of the incoming X-rays.

All in all, the picture on the film is thus a projection of the 3D attenuation distribution onto a 2D film. If you put a semi-transparent object on an overhead projector, you would get a somewhat similar situation.

Now, imagining that the film is digital and discrete. Thus, you should consider the film as an array of - let's say - 10 by 10 equal-sized detectors sitting right next to each other and forming a plate somewhat thicker than the film. The surface of each detector is quadratic. They cover an area of 10 cm by 10 cm.

Each square will integrate the energy of all the photons that it receives and produce *one* value, which will be the image value of the pixel it represents.

Problem 1

- On a separate piece of paper, draw (sketch) the detector array seen from above (physical sketch).
- Make another sketch (image sketch) showing the pixels of the corresponding image. We do not have an object, so you cannot sketch the image itself.

On the physical sketch, indicate the size of a detector. On the image, indicate the size a pixel represents.

We will next try to investigate which part of the object that contributes to a given pixel. As it is difficult to make a 3D drawing, we will make a copy of the physical sketch from (A), and then add two perpendicular frontal drawings as shown in Figure 2. On each of the frontal drawings, you will see

- the focal point of the X-ray tube at the top and
- the side of the detector array as a horizontal bar at the bottom of the drawing.

Problem 2

On the first drawing (the one with grey background in Figure 2), select a given detector in the 10 by 10 array of detectors. This detector produces a voltage that is represented by the associated pixel. Then mark the same detector's location on the two other drawings. Finally, on each of the three 2D-projections of the system, mark the volume that contributes to the pixel value for this particular detector (from focal point to detector).

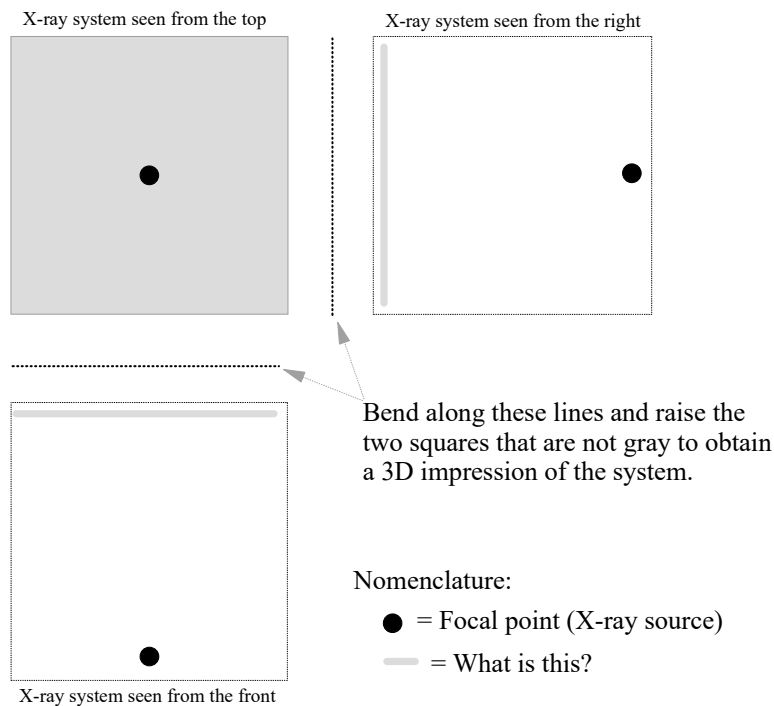


Figure 2 Layout showing the three drawings that you should use in Problem 2. Only the X-ray source is shown here. You should draw as many components as necessary to solve the problem.

What is the name of the *shape* this volume have? *Hint*: If a butterfly is flying around, try to identify the volume, in which, the butterfly will cast a shadow on the detector?

We will now consider the term *spatial resolution size*.

Problem 3

How would you define, or how would you measure, the spatial resolution size and what will it approximately be in our case? *Hints*: how many dimensions does the spatial resolution size have in our case? You should be able to arrive at - and provide - the result by drawing.

Problem 4

Now imagine that the detectors are made smaller, *e.g.*, the area of a detector is decreased to a quarter of the original one and the number of detectors are increased so as to cover the same physical area. What is the new dimension of the detector array? What happens to the spatial resolution size?

Problem 5

What gives the most detailed image, a large spatial resolution size or a small spatial resolution size? Consider how these terms are related to the terms high spatial resolution versus low spatial resolution. Are there any disadvantages by changing the size of the detectors?

Problem 6

Consider the final image: does it consist of pixels or voxels?

3 Tomographic imaging

If you do not have knowledge of computed tomography yet, please continue to Chapter 4.

We will now turn to *tomographic imaging*, *i.e.*, Computed Tomography (CT). In our situation, we will consider a simplified system as shown in Figure 3 which features an X-ray tube that sends out a narrow beam, *not* a fan beam as used by the scanner at the hospital. An analogy to this would be a laser diode (the X-ray tube) sending out a narrow beam towards a photo diode (detector).

Let us first consider the physics of the beam itself.

Problem 7

Draw the beam as if you look at it directly from the detector in Figure 3 (*i.e.*, in the analogy, you can consider using a laser pointer that point on a white wall (and never into the eyes of anyone)). Then draw an axis through a diameter of this circle. Let the axis be longer than the diameter. Give the axis a name (e.g. x -axis). Now make a sketch of the intensity of this beam as a function of distance along the axis you have defined. Remember to name both axis properly (also units); however, you do not need numbers on the axis. What shape will this function have? And why? Discuss different choices of shapes with your fellow students.

Problem 8

Now if this is the beam that is used to create a CT image, what is the contribution to the attenuation, when you consider both the shape of the beam and the shape and location of tissue structures? Consider for instance Figure 3b. Try to sketch this, maybe with several 2D drawings.

Next, we shall consider the image that comes out of the reconstruction. Remember that if we would like an image with four pixels (2 by 2), we need four projection values in order to solve four equations with four unknowns. If we require more pixels, we will need more projection values.

The 2D image from a CT scan consists of a number of pixels, say $N \times N$ pixels. The image is generated from a number of projections. The more pixels that are desired in the image, the larger are the number of projection values needed (which can be increased by increasing the number of projections and/or the number of detectors within the same over-all size of the detector).

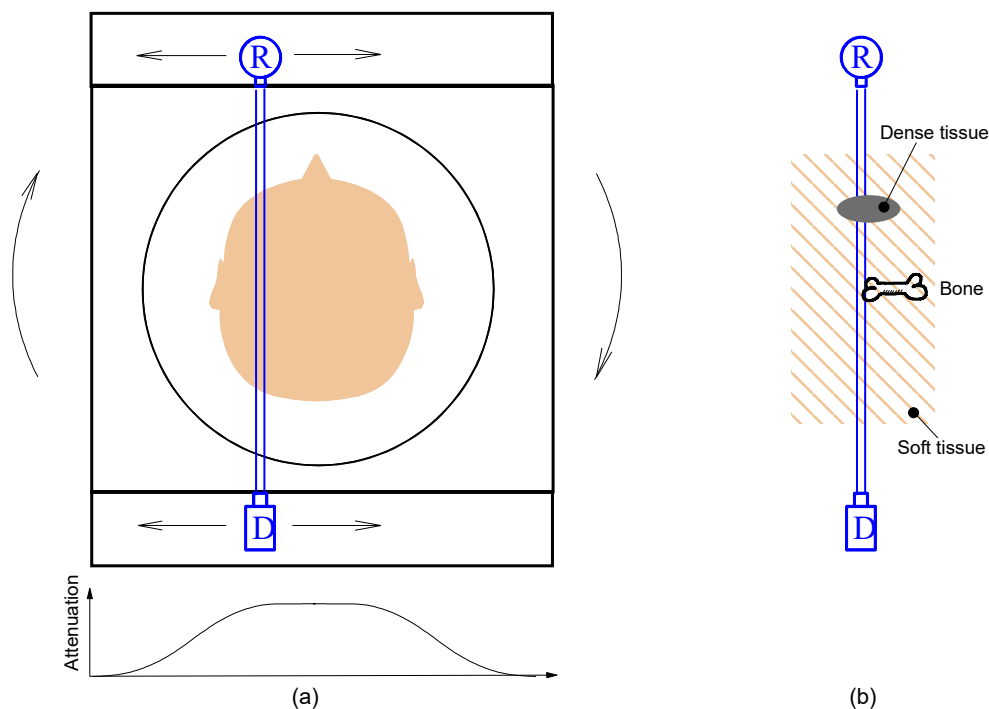


Figure 3 (a) Geometry of simple CT scanner. The x-ray tube (R) emits a thin beam. The X-ray tube is moved in unison with the detector (D). This is drawn in blue. (b) An example of tissues in the beam.

Now let us define the spatial resolution size: Imagine a point target (*e.g.* a lead sphere in air) that is imaged. The diameter is d . The diameter of the object that appears on the image, $\Delta d + d$, will always be larger than d . However, Δd might not be a constant, but will depend somewhat on d . The extra “size”, Δd , is a result of the fact that on any imaging system, an object will appear larger than it is. This “spreading” is described by the so-called *point spread function*, which is the “dot” you will see on an image, in case a very small object is imaged. We will set the width of this point spread function equal to the spatial resolution size.

Problem 9

If we increase the number of pixels in the recorded image, will the spatial resolution size decrease?
Will the diameter of the X-ray beam have any importance?

Problem 10

Does the image consist of pixels or voxels?

Problem 11

What is the third dimension of the spatial resolution size and what defines it?

Problem 12

Could we measure Δd by just using a infinitely small sphere? Hint: how much energy (or intensity) would be attenuated by something that is infinitely small? And what about the signal-to-noise ratio?

Problem 13

When dealing with 3D images, often the term *interslice distance* is used. What is this with respect to voxel size.

4 Appearance on planar X-ray

Consider the X-ray picture of Figure 4. The picture is taken in 2007 and represents one of the phantoms used then. It only shows the phantom itself including the acrylic box. On the right, there is a colour bar, but the unit that is supposed to be stated below the bar is marked with a “?”.

Problem 14

What is the unit of the image (*i.e.*, what should be stated, where the “?” is?) Are the values relative or absolute? And: what are the units of a CT image?

The next question concerns the appearance of the tube. Assumed that it is filled with air. A cross-sectional view is shown in Figure 5. The physical density of the agar is assumed to be that of water, while the physical density of the silicone rubber tube is assumed to be lower.

Problem 15

In this situation, draw the intensity profile as seen by the film. Also draw the same profile in the case where the density of the silicone rubber is assumed to be higher than that of water. In both cases, we assume that attenuation of X-rays varies directly with physical density.

Notice that you cannot be sure that the analysis you will do here will be identical to the one for your phantom. One of the aspects that could change is the content of the tube which might not always be air.

5 Acknowledgements

Mette L. V. Lauridsen is gratefully acknowledged for her help in re-formulating problems 1 to 6. Sara K. Møllenbach og Dorthe Bodholt Nielsen are gratefully acknowledged for their help in commenting

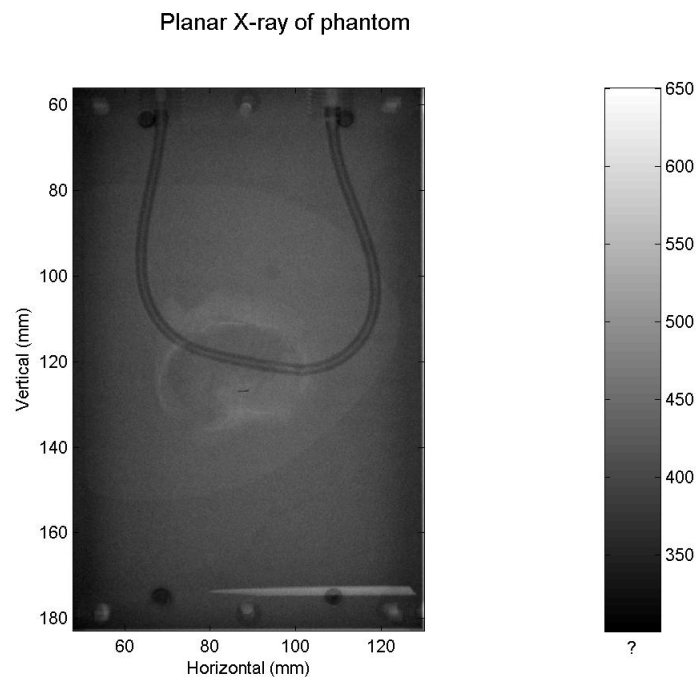


Figure 4 Planar X-ray image of one of the phantoms used in the course.

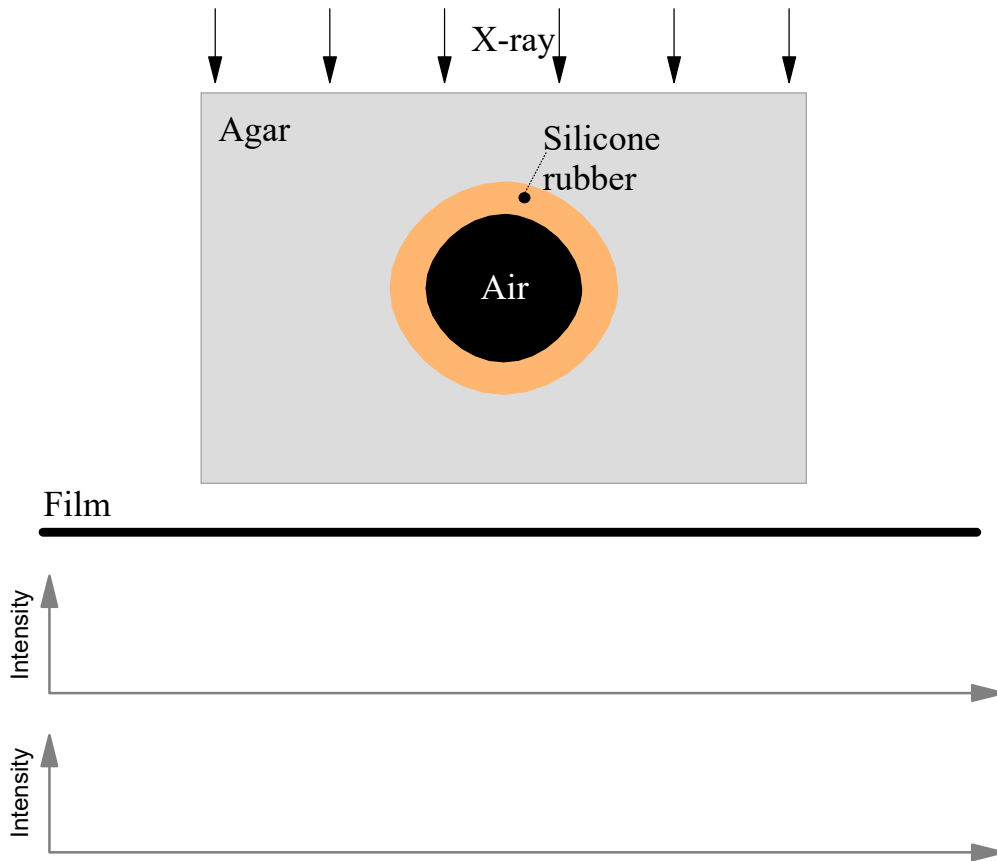


Figure 5 Illustration of the measurement situation. Under the film, you should draw the profile of the intensity of and proofreading this document. Finally, all the students that have asked questions have contributed as well!